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TRIGANOMETRIK MASALALARNI YECHISHDA QATOR EKVIVALENT NISBATLARNING TADBIQI

ANNOTATSIYA: Ushbu maqolada geometriyaning maktab kursida muhim rol o'ynaydigan trigonometriya bo'limi triganometrik masalalarni yechishga doir tadbiqlari qaralgan. Triganometrik masalalarni yechishda ba'zi elementini berib, uning qolgan elementlarini topish masalalari ko'rilgan. Bunday masalalarni yechishda trigonometriyadagi ekvivalent nisbatlarni qo'llab noma'lum elementlarni aniqlash usuli kabi amaliy masalalarni yechish qaralgan.
Kalit so'zlar: Sinuslar teoremasi, Kosinuslar teoremasi, Paskal masalasi, uchburchaklarni yechishda uch holdagi masalalar.

ABSTRACT: This article discusses the applications of geometry to solving trigonometric problems, a section of trigonometry that plays an important role in the school course. When solving trigonometric problems, the problems of finding the rest of its elements are considered, giving some element. Solving such problems involves solving practical problems, such as a method for determining unknown elements that support equivalent ratios in trigonometry.

Keywords: The sine theorem, the cosine theorem, Pascal's Problem, three-case problems in solving triangles.

KIRISH

Trigonometriyaning paydo bo'lishi amaliyotdagi hisoblashlar, aynan turli geometrik shakllar elementlarini topishda berilgan elementlar yetarli miqdori bo'yicha ushbu elementlarni aniqlash zarurati ehtiyoji bilan bog'liq. Antik davrdayoq Qadimgi Yunonistonda bir qator astronomik masalalar yechish bilan bog'liq hisob-kitoblar davomida trigonometriya sohasi rivojlanishiga muhim hissa qo'shildi. Trigonometriyaning shakllanishida X-IX va XIII asrlarda Markaziy Osiyo va Kavkazorti mintaqalari olimlarining ilmiy ishlari va yaratgan asarlari asosiy ahamiyatga ega.

Ilm-fanni keyingi rivojlanish davri shuni ko'rsatdiki, trigonometrik funksiyalar faqat ishlab chiqarishda emas, balki hisoblash geometriyasida yechish uchun zarur bo'lgan apparat vazifalarni; shuningdek, ushbu funksiyalar mexanika va fizikadagi davriy jarayonlarni o'rganishda ham muhim rol o'ynaydi. Shunday qilib, trigonometrik funksiyalar nazariyasiga asoslangan holda analitik geometriya yo'nalishi paydo bo'ldi. Trigonometrik funksiyalarning geometrik nazariyasi trigonometriyani amaliy masalalarga tadbiq qilish ko'proq mos keladi.

ASOSIY QISM

Masalaning mazmunidan kelib chiqishicha uchburchaklarni yechishda geometrik ko'rinishidan tashqari masalaning klassifikatsiyaga ega bo'lishi shubhasiz. Bular quyidagicha holatlarga bo'linadi:

a) birinchi tip - qandaydir ikki burchagi va bitta chiziqli element berilgan bo'lsin;
b) ikkinchi tip - qandaydir bitta burchagi va ikkita chiziqli element berilgan bo'lsin;
c) uchinchi tip - qandaydir uchta chiziqli element berilgan bo'lsin.

Trigonometrik masalalarni yechish metodi bo'yicha birinchi tipdagi masalalar bevosita qator ekvivalent nisbatlar vositalari bilan yechiladi. Ikkinchi tipdagi masalalar trigonometrik tenglamalar sistemasiga keltiriladi. Ushbu holatdagi masalalarda uchburchakning yana bir ikkinchi burchakni topish kerak bo‘ladi. Boshqacha aytganda, $\alpha+\beta+\gamma=\pi$ munosabat bajariladi. Uchinchi tipdagi masalalarda uchburchakning ikki burchagi topiladi. Umuman aytganda, shartda berilgan element uchburchak burchak elementi bo‘lmaganda, ya’ni uchburchak tomonlari berilganda masala soddalashadi.

1-masala. S, α va β berilgan. ρ va l_c ni toping.

a) bu masala birinchi tipdagi masaladir.

b) ρ ni aniqlash kerak. Qator teng nisbatdan ikki nisbatni olamiz, ya’ni birinchi nisbatda topish kerak bo‘lgan element, ikkinchisida berilgan elementda qatnashgan bo‘lsin:

$$\frac{\rho}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}=\sqrt{\frac{2S}{\sin\alpha\sin\beta\sin\gamma}}.$$

Bundan:

$$\rho=\sqrt{S\,\mathrm{ctg}\frac{\alpha}{2}\mathrm{ctg}\frac{\beta}{2}\mathrm{ctg}\frac{\gamma}{2}}\qquad[\gamma=\pi-(\alpha+\beta)];$$

b) l_c ni aniqlash kerak. Bunda ham yuqoridagi kabi l_c qatnashgan nisbat tanlaymiz:

$$\frac{l_c\cos\frac{\alpha-\beta}{2}}{\sin\alpha\sin\beta}=\sqrt{\frac{2S}{\sin\alpha\sin\beta\sin\gamma}}.$$

Bundan:

$$l_c=\sec\frac{\alpha-\beta}{2}\sqrt{\frac{2S\sin\alpha\sin\beta}{\sin\gamma}}.$$

2-masala. a, h_a va $\beta-\gamma=\varphi$ berilgan. α, β va γ ni aniqlang.

a) Bu masala ikkinchi tipdagi masaladir.

b) Chiziqli element qatnashgan qator teng nisbatlarni olamiz:

$$\frac{a}{\sin\alpha}=\frac{h_a}{\sin\beta\sin\gamma}=\cdots\qquad(A)$$

c) $\sin\beta\sin\gamma$ ni yig‘indiga ajratamiz:

$$\sin\beta\sin\gamma=\frac{1}{2}[\cos(\beta-\gamma)-\cos(\beta+\gamma)]=\frac{1}{2}[\cos\varphi-\cos(\beta+\gamma)].$$

Buni (A) ga qo‘yamiz :

$$\frac{a}{\sin\alpha}=\frac{2h_a}{\cos\varphi-\cos(\beta+\gamma)}.$$

$\beta+\gamma=\pi-\alpha$ bo‘lgani uchun $\frac{a}{2h_a}=\frac{\sin\alpha}{\cos\varphi-\cos\alpha}$.

Masala bir noma’lumli triganometrik tenglamaga aylandi. Bu yerda triganometrik funksiyalarni yarim argumentning tangensi orqali ifodalovchi formulalardan foydalanamiz:

$$\frac{a}{2h_a}=\frac{2\,\mathrm{tg}\frac{\alpha}{2}}{\cos\varphi\left(1+\mathrm{tg}^2\frac{\alpha}{2}\right)+1-\mathrm{tg}^2\frac{\alpha}{2}}.$$

Bundan

$$a\sin^2\frac{\varphi}{2}\mathrm{tg}^2\frac{\alpha}{2}+2h_a\,\mathrm{tg}\frac{\alpha}{2}-a\cos^2\frac{\gamma}{2}=0.$$

Demak,

$$\mathrm{tg}\frac{\alpha}{2}=\frac{-2h_a\pm\sqrt{4h_a^2+a^2\sin^2\varphi}}{2a\sin^2\frac{\varphi}{2}}.$$

Shu bilan birga ikkinchi yechim manfiy bo‘lganida o‘z-o‘zidan keraksiz bo‘lib qoladi. Shuning uchun $\frac{\alpha}{2}$ $\frac{\pi}{2}$ dan kichik. Demak,

$$\alpha=2\,\mathrm{arctg}\frac{-2h_a\pm\sqrt{4h_a^2+a^2\sin^2\varphi}}{2a\sin^2\frac{\varphi}{2}}.$$

d) $\beta+\gamma=\pi-\alpha$ dan hamda berilgan $\beta-\gamma=\varphi$ dan β va γ lar osongina topiladi.

3-masala. Paskal masalasi. a, α va $k=\frac{b-c}{h_a}$ berilgan. β va γ ni topish kerak.

a) bu masala birinchi tipga tegishli, $k-$ burchak elementi.

b) qator teng nisbatlardan:

$$\frac{b-c}{2\sin\frac{\alpha}{2}\sin\frac{\beta-\gamma}{2}}=\frac{h_a}{\sin\beta\sin\gamma}$$

yoki

$$\frac{2\sin\frac{\alpha}{2}\sin\frac{\beta-\gamma}{2}}{\sin\beta\sin\gamma}=k$$

ga ega bo‘lamiz, ya’ni tenglama uchta β, γ va $\frac{\beta-\gamma}{2}$ noma’lumga egadir.

c) shuning uchun oldingi masalalardagi kabi $\sin\beta\sin\gamma$ ko‘paytmani yig‘indiga aylantirib, soda to‘ldirishlar bilan ikki noma’lumli tenglamaga o‘tamiz:

$$k\cos(\beta-\gamma)-4\sin\frac{\alpha}{2}\sin\frac{\beta-\gamma}{2}+k\cos\alpha=0.$$

d) eng oxirida $\cos(\beta-\gamma)=1-2\sin^2\frac{\beta-\gamma}{2}$ bilan almashtirib, $\frac{\beta-\gamma}{2}$ noma’lumli tenglamaga kelamiz, unda

$$k-2k\sin^2\frac{\beta-\gamma}{2}-4\sin\frac{\alpha}{2}\sin\frac{\beta-\gamma}{2}+k\cos\alpha=0$$

yoki

$$k\sin^2\frac{\beta-\gamma}{2}+2\sin\frac{\alpha}{2}\sin\frac{\beta-\gamma}{2}-k\cos^2\frac{\alpha}{2}=0.$$

Bundan:

$$\sin\frac{\beta-\gamma}{2}=\frac{-\sin\frac{\alpha}{2}\pm\sqrt{\sin^2\frac{\alpha}{2}+k^2\cos^2\frac{\alpha}{2}}}{k}=\frac{1}{k}\sin\frac{\alpha}{2}\left(-1\pm\sqrt{1+k^2\,\mathrm{ctg}^2\frac{\alpha}{2}}\right).$$

Agar $k>0$ bo‘lsa, unda $\beta>\gamma$ bo‘ladi, shuning uchun olingan ikkala qiymatdan faqat musbat ishoralisini qoldiramiz. Bundan tashqari yordamchi burchak $\mathrm{tg}\,\varphi=k\,\mathrm{ctg}\frac{\alpha}{2}$ ni kiritib, radikal ishorasi ostida trivial ifoda hosil qilamiz.

Shunday qilib,

$$\sin\frac{\beta-\gamma}{2}=\frac{1}{k}\sin\frac{\alpha}{2}(\sec\varphi-1)=\frac{2\sin\frac{\alpha}{2}\sin^2\frac{\alpha}{2}}{k\cos\varphi}$$

yoki $\mathrm{tg}\,\varphi$ ni aniqlagandan so‘ng$k=\mathrm{tg}\,\varphi\,\mathrm{tg}\frac{\alpha}{2}$ ga ega bo‘lamiz. Unda

$$\sin\frac{\beta-\gamma}{2}=\frac{2\sin\frac{\alpha}{2}\sin^2\frac{\alpha}{2}}{\mathrm{tg}\,\varphi\cdot\mathrm{tg}\frac{\alpha}{2}\cdot\cos\varphi}=\cos\frac{\alpha}{2}\cdot\mathrm{tg}\frac{\varphi}{2}$$

bo‘ladi.

Bundan $\varphi=\mathrm{arc\,tg}\left(k\cdot\mathrm{ctg}\frac{\alpha}{2}\right)$ va $\beta-\gamma=2\arcsin\left(\cos\frac{\alpha}{2}\cdot\mathrm{tg}\frac{\varphi}{2}\right)$ kelib chiqadi.

e) $\beta-\gamma$ ni bilib va $\beta+\gamma=\pi-\alpha$ dan β va γ ni topish mumkin.

4-masala. h_a, r_a va 2ρ berilgan. α, β va γ ni toping.

a) Masala uchinchi tipga tegishli. Bunda biz ikkita triganometrik tenglama tuzib yechamiz va shu bilan ikkita burchakni aniqlaymiz.

b) Berilgan chiziqli ifodaning birinchi hadini trival had deb qarab, eng qulay nisbat

$$\frac{r_a}{2\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}} = \frac{h_a + r_a}{2\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\cos\frac{\beta-\gamma}{2}} = \frac{\rho}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}$$

yoki $\frac{h_a+r_a}{\cos\frac{\beta-\gamma}{2}} = \frac{\rho}{\cos\frac{\alpha}{2}} = \frac{r_a}{\sin\frac{\alpha}{2}}$ ni olish bilan bu ikki tenglamadan osongina maqsadga erishiladi:

$\mathrm{tg}\frac{\alpha}{2} = \frac{r_a}{\rho}$ va $\cos\frac{\beta-\gamma}{2} = \frac{(h_a+r_a)\sin\frac{\alpha}{2}}{r_a}$.

c) Bundan:

$$\alpha = 2\,\mathrm{arc\ tg}\frac{r_a}{\rho}, \qquad \beta-\gamma = 2\,\mathrm{arc}\ \cos\left[\left(\frac{h_a}{\rho}+1\right)\sin\frac{\alpha}{2}\right]$$

$$\beta+\gamma = \pi - \alpha$$

bo‘lgani uchun bulardan β va γ aniqlanadi.

Shu bilan birga masalani har vaqt yechib bo‘lmaydi. Faqatgina $\frac{h_a+r_a}{r_a}\sin\frac{\alpha}{2} < 1$ yoki $\frac{h_a+r_a}{r_a} < \mathrm{cosec}\frac{\alpha}{2}$ bo‘lganda yechish mumkin. $\mathrm{cosec}\frac{\alpha}{2}$ ni $\mathrm{tg}\frac{\alpha}{2} = \frac{r_a}{\rho}$ orqali ifodalab, $\mathrm{cosec}\frac{\alpha}{2} = \sqrt{1+\mathrm{ctg}^2\frac{\alpha}{2}} = \frac{\sqrt{r_a^2+\rho^2}}{r_a}$ ni olamiz va mumkin bo‘lgan shart $h_a + r_a < \sqrt{r_a^2+\rho^2}$ dan iborat.

XULOSA

Hozirgi kunda elementar matematika fanining alohida ajralmas qismi trigonometriyaning boshqa fanlar bilan o‘zaro bog‘liqligi, keng tarmoqlanganligi, fan va texnologiya rivojlanish taraqqiyotida har qadamda trigonometriyaga duch kelamiz. Ya’ni matematikadan tashqari boshqa fanlarni o‘rganishda ushbu fan bo‘limini chuqur bilishni talab qilmoqda.

Trigonometriya fan bo‘limini har tomonlama va chuqur o‘rganish, buy yo‘nalishdagi qo‘shimcha ilmiy-metodik materiallar orqali matematik bilim va ko‘nikmalarni kengaytirish maqsadga muvofiq bo‘ladi.

Geometriyaning maktab kursidagi boshqa sohalari kabi trigonometriyada ham nazariyani amalda tadbiq etish, ya’ni masalalar yechish malakasini orttirish talab etiladi. Ammo amaliyotda bu sohada ko‘pgina qiyinchiliklarga duch kelinadi.

Shu kabi amaliy masalalarda ham trigometrik masalalarni yechishda qator ekvivalent nisbatlarning tadbiq etish muhim rol o‘ynaydi.

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