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MONOTONOUS AND BOUNDED SEQUENCES

Abstract. Monotonic sequence is one of the simplest terms used in mathematics to refer to a number sequence that moves from a smaller value to a bigger value or vice versa; that is, it only increases or decreases. Different fields of study where this type of sequence is important include calculus, probability and computer science. Mastering monotonically increasing and decreasing sequences is particularly important for studying the convergence and behavior of mathematical functions and series.

Key words: monotone sequences, percentage, theorem, numerical sequences, mathematical properties, etc.

The main problem in the study of numerical sequences is the problem of the existence of their limit. Although solving this problem in general is quite complicated, it is relatively easy to solve for some classes of Ie kin sequences. Especially for monotone sequences, the limit existence problem has a simple solution. We call increasing sequences and decreasing sequences monotonic sequences. Sometimes, strictly increasing and strictly decreasing sequences are called strictly monotonic sequences. Note that according to the above definitions, a stationary sequence is both increasing and decreasing. The peculiarity of this example is that its terms oscillate around zero with constant amplitude, sometimes increasing and sometimes decreasing. In other words, this sequence is not monotonic. Interestingly, if the sequence is monotone, it is necessary and sufficient that it converges. Thus, if we want to demand the boundedness of a monotone sequence, it is enough to demand that it is bounded from above when it is increasing (since it is also bounded from below), and bounded from below when it is decreasing.

Monotonicity and boundedness are two very strong characteristics in the context of testing convergence and divergence of real sequences. We have a very important theorem, namely, the monotone convergence theorem, important in the sense of its usefulness and application, which not only ensures us that a bounded monotonic sequence of real numbers is always convergent, but also unambiguously points out the fact that such a sequence always converges to its lub (least upper bound or supremum)/glb (greatest lower bound or infimum) depending on the monotonicity nature of the particular sequence. So, it is considered as a strong tool of analysis for handling the problem of testing convergence/divergence of monotonic sequences. In this discourse, we will mainly discuss about monotonic sequences and the very useful monotonic convergence theorem in detail. We will also deal with some particular problems where monotone convergence theorem can be applied very efficiently.

Definition.1. A real sequence $X = \langle x_n \rangle$ is said to be

i) An increasing sequence if $x_1 < x_2 < x_3 < .... < x_n < ....$ i.e., if $x_n < x_{n+1}, \forall n \in \mathrm{N}$.

ii) A decreasing sequence if $x_1 > x_2 > x_3 > .... > x_n > ....$ i.e., if $x_n > x_{n+1}, \forall n \in \mathrm{N}$.

iii) A monotonic increasing sequence if $x_1 \le x_2 \le x_3 \le x_4 \le .... \le x_n \le ....$ i.e., if $x_n \le x_{n+1}, \forall n \in \mathrm{N}$.

iv) A monotonic decreasing sequence if $x_1 \ge x_2 \ge x_3 \ge x_4 \ge .... \ge x_n \ge ....$ i.e., if $x_n \ge x_{n+1}, \forall n \in \mathrm{N}$.

v) A monotonic sequence if it is either monotonic increasing or monotonic decreasing.

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The monotone convergence theorem confirms that there is a limit for every bounded monotone sequence. It also gives us a way of calculating the limit of the sequence provided we can evaluate the supremum (in case of monotonic increasing sequence)/ infimum (in case of monotonic increasing sequence). Sometimes it is very much difficult to evaluate the limit. But if we know that the limit exists, then we can somehow evaluate it.

Look at the sequence of numbers: 1, 2, 4, 8, 16, 0. . . This sequence is increasingly monotonic as is given by the fact that each element of the sequence is two times of previous element of the sequence. This preset sequence can be presented graphically on a coordinate plane to represent the terms. What this means is that a set of points will be established such that when the curve defined by these points is created, the entire curve will lie wholly in the first quadrant of the X-Y axis and will not extend downward.

Example. 1.: $\lim\left(\frac{1}{\sqrt{n}}\right)=0$

Here, $n \geq 1, \forall n \in \mathrm{N} \Rightarrow \sqrt{n} \geq 1$ *and hence* $\frac{1}{\sqrt{n}} \leq 1$. So, $\left\langle \frac{1}{\sqrt{n}} \right\rangle$ is bounded.

Further,

$$n+1>n, \quad \forall n \in \mathrm{N} \Rightarrow \sqrt{n+1} \geq \sqrt{n}$$
$$\Rightarrow \frac{1}{\sqrt{n}} \geq \frac{1}{\sqrt{n+1}}$$

Inductively defined sequences are treated differently as the limits of such sequences cannot be evaluated so much easily. If we know that such sequences are convergent, then, in most of the cases the limits are calculated by using the inductive relations. As for subsets of R, there is a concept of boundedness for sequences. Basically a sequence is bounded (or bounded above or bounded below) if the set of its terms, considered as a subset of R, is bounded.

For instance, let us take the example of the increasing sequence, { 1, 1/2, 1/4, 1/8, . . . } This is a decreasing sequence, so it appears to be monotonic, and since it is also bounded above by 1, it converges to 0. The fact that the sequence is bounded implies that the terms of the sequence cannot diverge to infinity, while it's being monotonic implies that the sequence is either strictly increasing or strictly decreasing, thus it has to converge. Monotonic sequences can be compared with other kinds of sequences, like arithmetic sequences, geometrical sequences, and Fibonacci sequences. All in all, there are certain similarities between these types of sequences, but each has distinct features and characteristics of its own.

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