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TEBRANISH TENGLAMALARINI UMUMIY YECHIMINI TOPISH VA ULARNI MAPLE PAKETI ORQALI TASVIRLASH

Haydarov To'lqinjon Turg'unboyevich

Jizzax politexnika instituti "Oliy matematika" kafedrasi katta o'qituvchisi

ANNOTATSIYA: Tebranish tenglamalarini umumiy yechimini topish va ularni Maple paketi yordamida yechish jarayonini qiziqarli misollar yordamida tasvirlash. Maple paketi orqali yechilgan misollarning, shuningdek, yechimning ikki o'lchovli animatsiyali grafiglarini tasvirlash.

KALIT SO'ZLAR: to'lqin tarqalish tenglamalari, Koshi masalasi, Dalamber formulasi, boshlang'ich shartlar, chegaraviy shartlar, umumiy yechim, yechim grafigi.

Bir jinsli to'lqin tenglamasini

$$u_{tt} = a^2 u_{xx}$$

boshlang'ich shartlarni

$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x)$

qanoatlantiruvchi yechimi quyidagi Dalamber formulasi orqali topiladi:

$$u(x,t) = \frac{1}{2}\big(u_0(x-at) + u_0(x+at)\big) + \frac{1}{2a}\int_{x-at}^{x+at} u_1(s)ds$$

bu yerda $u_0(x)$, $u_1(x)$ berilgan funksiyalar.

Bir jinsli bo'lmagan to'lqin tenglamasining

$$u_{tt} = a^2 u_{xx} + f(x,t)$$

boshlang'ich shartlarni

$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x)$

qanoatlantiruvchi yechimi quyidagi Dalamber formulasi orqali topiladi:

$$u(x,t) = \frac{1}{2}\big(u_0(x-at) + u_0(x+at)\big) + \frac{1}{2a}\int_{x-at}^{x+at} u_1(s)ds + \frac{1}{2a}\int_0^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} f(s,\tau)dsd\tau$$

Bu yerda $u_0(x)$, $u_1(x)$, $f(x,t)$ berilgan funksiyalar.

1-misol. $u_{tt} = 4u_{xx}$ tenglamaning $u(x,0) = x^2$, $u_t(x,0) = x$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

Yechish. Yuqoridagi masalani yechishda bir jinsli tor tebranish tenglamasi uchun Dalamber formulasidan foydalanamiz:

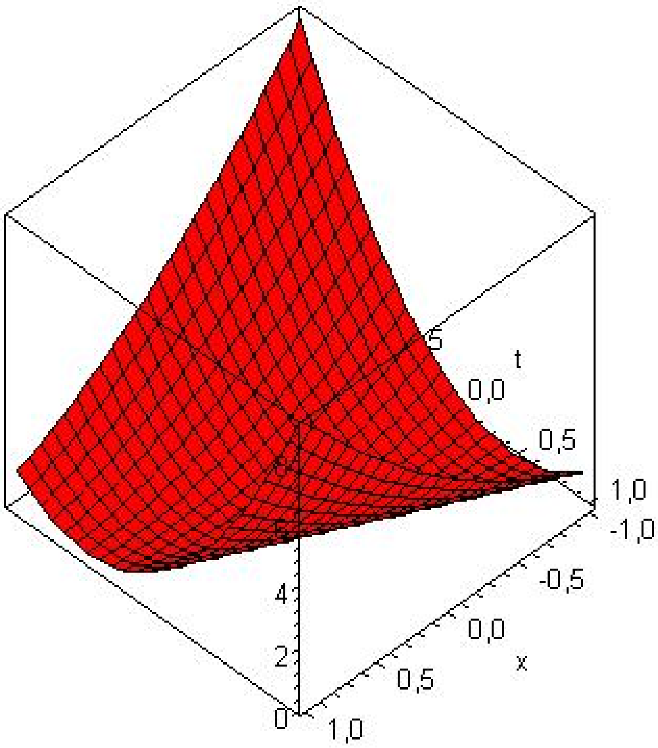
$$u(x,t) = \frac{1}{2}[(x-2t)^2 + (x+2t)^2] + \frac{1}{4}\int_{x-2t}^{x+2t} sds = x^2 + 4t^2 + 4xt$$

Endi yechimning grafigini Maple paketida ko'rinishini keltiramiz:

> $u(x, t) := x^2 + 4 \cdot t^2 + 4 \cdot x \cdot t;$

$u := (x, t) \to x^2 + 4\,t^2 + 4\,t\,x$

> **plot3d**(u(x, t), **x = −1 ..1, t = −1 ..1, color = red**);



2-misol. $u_{tt} = 4u_{xx} + e^x + t$ tenglamaning $u(x,0) = x$, $u_t(x,0) = \frac{\ln(x)}{x}$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini toping.

Yechish: Bir jinslimas tor tebranish tenglamasi uchun Dalamber formulasidan foydalanamiz

$$u(x,t) = \frac{1}{2}[(x-2t)+(x+2t)] + \frac{1}{4}\int_{x-2t}^{x+2t} \frac{\ln(s)}{s} ds + \frac{1}{4}\int_0^t \int_{x-2(t-\tau)}^{x+2(t-\tau)} (e^s + \tau) ds d\tau =$$

$$= x + \frac{1}{8}[ln^2(x+2t) - ln^2(x-2t)] - \frac{1}{4}e^x + \frac{1}{6}t^3 + \frac{1}{8}(e^{x+2t} + e^{x-2t})$$

Topilgan umumiy yechimning grafigini Maple paketi yordamida chizamiz:

> $u(x, t) := x + \frac{1}{8} \cdot \left(\ln^2(x + 2\,t) - \ln^2(x - 2\,t)\right) - \frac{1}{4} \cdot \exp(x) + \frac{1}{6} \cdot t^3 + \frac{1}{8} \cdot (\exp(x + 2\,t) + \exp(x - 2\,t));$

$$u := (x,t) \to x + \frac{1}{8}\ln(x+2t)^2 - \frac{1}{8}\ln(x-2t)^2 - \frac{1}{4}e^x + \frac{1}{6}t^3 + \frac{1}{8}e^{x+2t} + \frac{1}{8}e^{x-2t}$$

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> plot3d(u(x, t), x = -10 ..10, t = -10 ..10,
    color = red);
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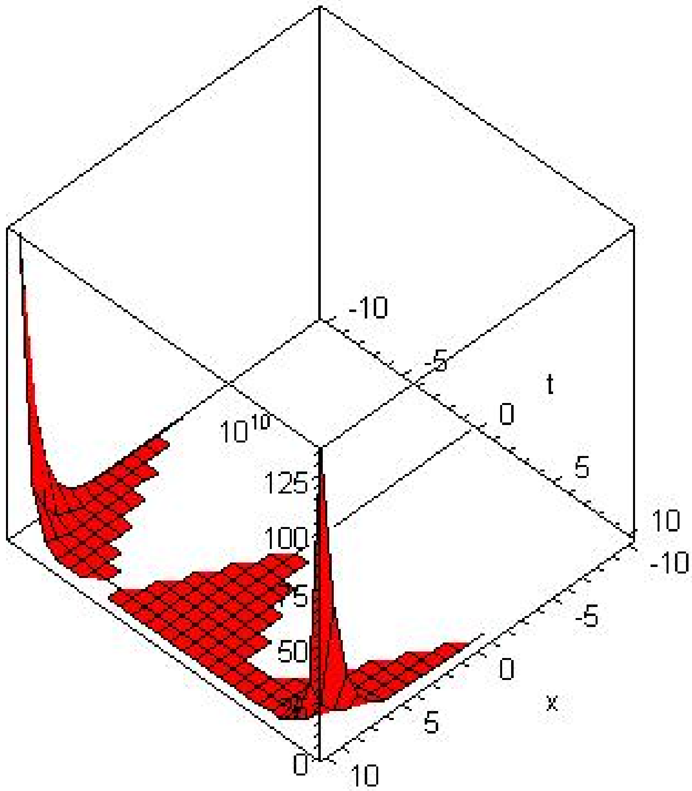

Foydalanilgan adabiyotlar

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