ADVANTAGES OF SOLVING A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS IN SIMPLE ITERATION AND ZEYDEL METHODS

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Annotation: The solutions of the system of linear algebraic equations cannot always be found in exact ways. Therefore, approximate calculation methods are used. In this work, the Seidel and iteration methods of solving the system of differential equations are considered. The program is shown in the example and the results are shown in the program.

The system of linear algebraic equations can be solved by exact and approximate (iterative) methods. The advantage of solving the system of linear algebraic equations by the iterative method, that is, by the method of successive approximation, is the simplicity of the application of such solving methods to programming. Iterative methods require initial approximation to the desired solution before calculation. The speed of convergence of the iterative process depends on the choice of the initial approximation, and also the iteration method allows to find the solution of the system of linear algebraic equations with a given accuracy.

To use the iterative method

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ &\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n \end{aligned} \qquad (1)$$

system of equations

$$\bar{x} = G\bar{x} + \bar{f} \qquad (2)$$

iterative process by bringing it into view

$$\bar{x}^{(k+1)} = G\bar{x}^{(k)} + \bar{f}\ , \ \text{k} = 0, 1, 2, \dots \ (3)$$

performed by recurring formulas. Matrix G and vector are formed as a result of substitution in the system of linear algebraic equations above. ‖ ... ‖ symbol denotes the norm of the matrix. ‖G‖ of the matrix norm < 1 (3) is a necessary and sufficient condition for the approximation of equality.

Also ,$|a_{ii}| \ > \overset{n}{\underset{i,j=1;i\quad j}{}} |a_{ij}|, \qquad A = \{a_{ij}\}_1^n$ (4)

convergence is guaranteed when the condition is met.

$\| \bar{x}^* - \bar{x}^{(k+1)} \| \qquad \|G\|^{k+1} \ \| \bar{x}^{(0)} \| + \frac{\|G\|^{k+1}}{1-\|G\|} \ \| \bar{f} \|$ condition is a condition for evaluating the absolute error of a simple iteration.

Consider the following example:

Using simple iteration and Seidel methods, solve the system of equations with Ɛ=0.001 accuracy:

$$\begin{cases} 8x_1 + x_2 + 9x_3 = 10 & (I) \\ x_1 + 2x_2 + 6x_3 = 2 & (II) \\ -2x_1 + 5x_2 + 2x_3 = 9 & (III) \end{cases}$$

Condition (4) is not fulfilled for the system, therefore, we make it appear in accordance with the given requirements, that is, we make it so that the coefficients of the main diagonal are greater than the sum of the coefficients in front of the remaining variables of the line, for this, we perform the following steps in a row we do: $\begin{cases} 7x_1 - x_2 + 3x_3 = 8 & (I)-(II) \\ -2x_1 + 5x_2 + 2x_3 = 9 & (III) \\ x_1 + 2x_2 + 6x_3 = 2 & (II) \end{cases}$

$$\begin{cases} x_1 = \frac{1}{7}(8 + x_2 - 3x_3) \\ x_2 = \frac{1}{5}(9 + 2x_1 - 2x_3) \\ x_3 = \frac{1}{6}(2 - x_1 - 2x_2) \end{cases} => \begin{cases} x_1 = 1{,}143 + 0{,}143x_2 - 0{,}429x_3 \\ x_2 = 1{,}8 + 0{,}4x_1 - 0{,}4x_3 \\ x_3 = 0.333 - 0{,}167x_1 - 0.333x_2 \end{cases} \quad (5)$$

Add the coefficients in front of the unknowns on the right-hand side of each equationz:

0,143+0,429=0.572; 0,4+0,4=0,8; 0,167+0,333=0,5

The maximum number of steps that gives the solution with an accuracy of Ɛ = 0.001 can be found by the following relation:

$$\|X^0 - X^k\| \le \frac{\|A\|^{k+1}}{1-\|A\|} * \|B\| < \varepsilon$$

(We evaluate the approximation from the formula (4). Here $\|A\|_1 = \max(0{,}572; 0{,}8; 0{,}5) = 0{,}8 < 1$; ; we find the norm of the vector matrix formed from the free terms of the three equations: $\|B\|_1 = \max(1{,}143; 1{,}8; 0{,}2) = 1{,}8$.. The values found $\|X^0 - X^k\| \le \frac{\|A\|^{k+1}}{1-\|A\|} * \|B\| < 0.001$ put in the formula and evaluate k:$\frac{0{,}8^{k+1}}{0{,}2} * 1{,}8 < 0{,}001$; $0{,}8^{k+1} < \frac{0{,}001*0{,}2}{1{,}8}$;

$(k+1)\lg 0,8 < -3 + \lg 0,2 - \lg 1,8$

$(k+1)(-0.097) < -3 - 0,699 + 0,255; \quad k+1 > 40,8; \quad k \geq 40$

So, rapprochement is ensured.

As an initial approximate value, we take a vector of free terms, i.e $\bar{x}^{(0)} = (1,143; 1,800; 0,333)^T$.

$\bar{x}^{(0)}$ put their values into equation (5):

$$\begin{cases} x_1 = 1,143 + 0,143 * 1,8 - 0,429 * 0,333 \\ x_2 = 1,8 + 0,4 * 1,143 - 0,4 * 0,333 \\ x_3 = 0.333 - 0,167 * 1,143 - 0.333 * 1,8 \end{cases}$$

Continuing the calculation, we enter the results into the table:

k	x_1	x_2	x_3
0	1,143	1,800	0,333
1	1,2571	2,1238	-0,4571
2	1,6422	2,4857	-0,5841
3	1,7483	2,6905	-0,7689
4	1,8568	2,8069	-0,8549
5	1,9102	2,8847	-0,9118
6	1,9457	2,9288	-0,9466
7	1,9669	2,9569	-0,9672
8	1,9798	2,9737	-0,9801
9	1,9877	2,9840	-0,9879
10	1,9925	2,9902	-0,9926
11	1,9954	2,9940	-0,9955
12	1,9972	2,9964	-0,9973
13	1,9983	2,9978	-0,9983

Answer: $X^{(13)} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$.

$$\begin{cases} x_1 = 0,42x_1 - 0,14x_2 + 0,31x_3 + 0,58 \\ x_2 = -0,18x_1 - 0,57x_2 + 0,12x_3 - 0,12 \\ x_3 = 0,06x_1 - 0,06x_2 + 0,05x_3 - 0,35 \end{cases}$$

Continuing the calculation, we enter the results into the table: Since the norm of the matrix composed of the coefficients in front of the unknowns on the right side of the equations is $\|A\|_1 = \max(0,572; 0,8; 0,5) = 0,8 < 1$; , Zeidel the iterative process approaches. We place the calculation results in the following table:

k	x_1	x_2	x_3
0	1,143	1,800	0,333
1	1,2571	2,1695	-0,5994
2	1,7097	2,7236	-0,8595
3	1,9003	2,9039	-0,9514
4	1,9654	2,9667	-0,9831
5	1,9880	2,9885	-0,9942
6	1,9958	2,9960	-0,9980
7	1,9986	2,9986	-0,9993

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8	1,9995	2,9995	-0,9998

If it is required to solve one system by simple iteration and Seidel methods, it is preferable to use the Seidel method. Because if the condition (4) is satisfied, the Seidel iteration converges faster than the simple iteration.

Solutions of systems of linear algebraic equations cannot always be found in exact ways. Therefore, approximate calculation methods are used.

```
import sys

limit = int(input("Enter the Total Number of Equations:\t"))
# maximum error limit till which errors are considered,
# or desired accuracy is obtained)
allowed_error = float(input("Enter Allowed Error:\t"))
print("\nEnter the Co-Efficients\n")

matrix = [[0 for j in range(limit+1)] for i in range(limit)]
y = [0 for i in range(limit)]

# Read in matrix of coefficients and constants
for count in range(limit):
    for t in range(limit+1):
        print(f" Matrix[{count+1}][{t+1}] = ", end='')
        matrix[count][t] = float(input())

# Initialize the solution vector
for count in range(limit):
    y[count] = 0

# Perform Gauss-Jordan elimination
while True:
    error = 0
    for count in range(limit):
        temp = matrix[count][limit]
        for t in range(limit):
            if t != count:
                temp -= matrix[count][t] * y[t]
        temp /= matrix[count][count]
        if abs(temp - y[count]) > error:
            error = abs(temp - y[count])
        y[count] = temp
        print(f"\nY[{count+1}]:\t{y[count]}")
    if error < allowed_error:
        break

# Print the solution vector
print("\n\nSolution\n\n")
for count in range(limit):
    print(f" \nY[{count+1}]:\t{y[count]}")
```

```
Y[1]:    -3.7326605402543 01e+265
Y[2]:    -8.652427335233774e+265
Y[3]:    1.789840779783 0132e+266
Y[1]:    -1.905415553556546 8e+266
Y[2]:    -4.416814571566305e+266
Y[3]:    9.136620893350296e+266
Y[1]:    -9.726596683573294e+266
Y[2]:    -2.254656433826424e+267
Y[3]:    4.663981416208 73e+267
Y[1]:    -4.965147039006517e+267
Y[2]:    -1.150937072912293e+268
Y[3]:    2.380827978380080 8e+268
Y[1]:    -2.534564341563554e+268
Y[2]:    -5.875201764358465e+268
Y[3]:    1.215344006933261e+269
Y[1]:    -1.293821985745437 8e+269
```

We compared the results of the program with the results in Excel. We think that this work will be very useful for students and teachers. Because in the work, the system of algebraic linear equations is developed both analytically and by software. It will be much easier for students to understand.

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