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IRRATSIONAL TENGLAMALARNI YECHISHNING NOSTANDART USULLARI

Annotatsiya: Ushbu maqolada tenglamalar yechishning nostandart usullari bilan tanishtirib, ularni yechish usullari ko ‘rsatib berilgan. Hamda bir nechta tenglamalar yechish usullari tavsiya qilingan.

Kalit so ‘zlar: Irratsional, parametr, ratsional son, tenglama, Bezu teoremasi.

Kirish

Tenglamalarni nostandart usulda yechish o ‘quvchilar bilimini oshirish bilan birgalikda har qanday masalalarni tez yechishni topishga yordam beradi.

Ko ‘pgina tenglamalarni, ayniqsa, ikkinchi va undan yuqori darajali tenglamalar sistemalarini odatdagi usullar bilan, masalan, o ‘rniga qo ‘yish usuli yordamida yechish qo ‘pol, uzoq shakl almashtirishlarga olib kelib natijani hosil qilib bo ‘lamaslikka sabab bo ‘ladi, vaholanki, bu tenglamalarni ancha ratsional, ya’ni boshqacha aytadigan bo ‘lsak, nostandart usullar bilan osongina yechish mumkin.

1. $2x^2 + 3x - 5\sqrt{2x^2 + 3x + 9} + 3 = 0$ tenglamani yechamiz

Yechish: Agar bu tenglamani irratsional tenglama sifatida odatdagi usul bilan yechadigan bo ‘lsak, to ‘rtinchi darajali tenglama hosil bo ‘lib, uni Bezu teoremasini bilmay turib yecha olmaymiz. Ko ‘pincha hosil bo ‘lgan tenglama, masalan, irratsional ya’ni kasr ildizlarga ega bo ‘lsa, uni yechishda, Bezu teoremasi ham yordam bera olmaydi. O ‘zgaruvchi qatnashgan qo ‘shiluvchilar ildiz ostida ham undan tashqarida ham bir xil ekaniga e’tibor beraylik. Yangi

$y = \sqrt{2x^2 + 3x + 9}, y \geq 0$ o ‘zgaruvchi kiritamiz. U holda berilgan tenglama $y^2 - 9 - 5y + 3 = 0$ ko ‘rinishga ega bo ‘lib, bu yerdan $y = -1$, ya’ni $y = 6$ ga egamiz, biroq, $y \geq 0$, demak, $\sqrt{2x^2 + 3x + 9} = 6$. Bu tenglamani yechib,

$x = -4{,}5$ ya’ni $x = -3$ ni topamiz. Tekshirish orqali o ‘zgaruvchining bu qiymatlari berilgan tenglamani qanoatlantirishga ishonch hosil qilamiz. Ko ‘rib chiqilgan usul hatto harfiy parametrga ega bo ‘lgan shunday ko ‘rinishdagi tenglamalar uchun ham qo ‘llanilishi mumkin. Biroq, harfiy parametrlar qatnashgan holda bu parametrlar mazkur tenglamada qanday qiymatlar qabul qila

olishi mumkinligini tekshirish kerak.

2. $\sqrt{12 + \sqrt{12 + x}} = x$ tenglamani yechamiz

Yechish : $x > 0$ ekani ma’lum. Yangi $y = \sqrt{12 + x}$ o ‘zgaruvchi kiritamiz bu yerda $y > 0$. U holda $12 + y = x^2$ ga ega bo ‘lamiz. Quyidagi tenglamalar sistemasiga ega bo ‘lamiz:
$$\begin{cases} y^2 - x = 12 \\ x^2 - y = 12 \end{cases}$$

Birinchi tenglamadan ikkinchi tenglamani ayirsak $y^2-x^2+y-x=0$ ni hosil qilamiz. Ko ‘paytuvchilarga ajratsak $(y-x)(y+x+1)=0$. Ma'lumki,

$x>0; y>0$, shuning uchun $x+y+1>0$. Demak, $y-x=0$, ya'ni $y=x$. Buni ikkinchi tenglamaga qo ‘ysak $x^2-x-12=0$ hosil bo ‘ladi. Bu tenglama ildizlari $x=4$ ya'ni $x=-3<0$, demak, $x=4$ tenglamaning ildizi ekan.

3. Ushbu $4\sqrt[n]{(2x-7)^2}+\sqrt[n]{(2x-7)^2}=\sqrt[n]{4x^2-49}$ tenglamani yechamiz.

Yechish: $x=3{,}5$ tenglamani qanoatlantirmaydi. Demak, biz tenglamani yechimini $x\neq 3{,}5$ qiymatlar to ‘plamida qarashimiz mumkin. Tenglamaning ikkala qismini $\sqrt[n]{(2x-7)^2}$ ga bo ‘lib,

$$4+\sqrt[n]{\left(\frac{2x+7}{2x-7}\right)^2}=4\sqrt[n]{\frac{2x+7}{2x-7}}$$

ga ega bo ‘lamiz. Yangi $y=\sqrt[n]{\frac{2x+7}{2x-7}}$ o ‘zgaruvchi kiritamiz, bu yerda $y\geq 0$. Natijada $y^2-4y+4=0$ tenglamaga ega bo ‘lamiz bu tenglamani ildiz $y=2$

bo ‘ladi.

Demak, $\sqrt[n]{\frac{2x+7}{2x-7}}=2$, ya'ni $\frac{2x+7}{2x-7}=2^n$. Bundan $x=\frac{7(2^n+1)}{2(2^n-1)}$ ga ega bo ‘lamiz.

4. Ushbu $2a\sqrt[2pq]{x^{p+q}}=\sqrt[p]{x}+\sqrt[q]{x}$ tenglamani yechamiz.

Yechish: Radikallarni kasr ko ‘rsatkichli darajalar bilan almashtiramiz va hamma hadlarni chap tomonga o ‘tkazamiz:

$2ax^{\frac{p+q}{2pq}}-x^{\frac{1}{p}}-x^{\frac{1}{q}}=0$ ni hosil qilamiz. Barcha qo`shiluvchilarda x qatnashadi. Bunday hollarda x o ‘zgaruvchining eng past darajasini qavsdan tashqariga chiqarish kerak. Ko ‘rilayotgan misolda $x^{\frac{1}{p}}$ yoki $x^{\frac{1}{q}}$ ni qavsdan tashqariga chiqaramiz. Natijada tenglama

$$x^{\frac{1}{q}}\left(2ax^{\frac{q-p}{2pq}}-x^{\frac{q-p}{pq}}-1\right)=0$$

ko ‘rinishga ega bo ‘ladi, bu yerdan $x^{\frac{1}{q}}=0$, ya'ni $x=0$ yoki

$2ax^{\frac{q-p}{2pq}}-x^{\frac{q-p}{pq}}-1=0$. Yangi $y=x^{\frac{q-p}{2pq}}$ o ‘zgaruvchi kiritamiz, u holda

$2ay-y^2-1=0$, ya'ni $y^2-2ay+1=0$, bu yerdan $y=a\pm\sqrt{a^2-1}$, bu yerda $|a|\geq 1$, demak, $x^{\frac{q-p}{pq}}=a\pm\sqrt{a^2-1}$ .ya'ni $x=\left(a\pm\sqrt{a^2-1}\right)^{\frac{2pq}{q-p}}$, shunday qilib, $x=0$ yoki $x=\left(a+\sqrt{a^2-1}\right)^{\frac{2pq}{q-p}}$ yoki $x=\left(a-\sqrt{a^2-1}\right)^{\frac{2pq}{q-p}}$ bu yerda $|a|\geq 1$

5. Ushbu $\sqrt[n]{x^n + \sqrt[n+1]{a^n x^2}} + \sqrt[n]{a^n + \sqrt[n+1]{a^{n^2} x^n}} = b$ tenglamani yechamiz.

Yechish: Radikallarni kasr ko ‘rsatkichli darajalar bilan almashtiramiz, so ‘ngra chap tomonnni ko ‘paytuvchilarga ajratamiz:

$$\sqrt[n]{x^{\frac{n^2}{n+1}}\left(x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}}\right)} + \sqrt[n]{a^{\frac{n^2}{n+1}}\left(a^{\frac{n}{n+1}} + x^{\frac{n}{n+1}}\right)} = b,$$

$$x^{\frac{n}{n+1}}\left(x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}}\right)^{\frac{1}{n}} + a^{\frac{n}{n+1}}\left(a^{\frac{n}{n+1}} + x^{\frac{n}{n+1}}\right)^{\frac{1}{n}} = b,$$

$$\left(x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}}\right)^{\frac{1}{n}}\left(x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}}\right) = b$$

$$\left(x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}}\right)^{\frac{n+1}{n}} = b.$$

Oxirgi tenglamalarning ikkala tomonini $\frac{n}{n+1}$ – darajaga ko ‘tarib, quyidagini hosil qilamiz:

$$x^{\frac{n}{n+1}} + a^{\frac{n}{n+1}} = b^{n+1}.$$

Bu yerdan $x^{\frac{n}{n+1}} = b^{\frac{n}{n+1}} - a^{\frac{n}{n+1}},\ x = \left(b^{\frac{n}{n+1}} - a^{\frac{n}{n+1}}\right)^{\frac{n+1}{n}}.$

Tenglama $b \geq a > 0$ bo ‘lganda yechimga ega bo ‘ladi.

Mustaqil yechish uchun masalalar

1. Tenglamani yeching: $x^2 + 2\sqrt{x^2 - 3x + 11} = 3x + 4$
2. Tenglamani yeching: $\sqrt[n]{(x+1)^2} + 3\sqrt[n]{(x-1)^2} = 4\sqrt[n]{x^2 - 1}$
3. Tenglamani yeching: $\sqrt[4]{x(x+5)^2} + 6\sqrt[4]{x^3} = 5\sqrt[4]{x^2(x+5)}$
4. Tenglamani yeching: $\sqrt{a - \sqrt{a + x}} = x$
5. Tenglamani yeching: $\left(\sqrt{2+\sqrt{3}}\right)^x + \left(\sqrt{2-\sqrt{3}}\right)^x = 4$
6. $\sqrt{x - 2\sqrt{x-1}} = 1 + \sqrt{x + 3 - 4\sqrt{x-1}}$
7. $\sqrt{x^2 - p} + 2\sqrt{x^2 - 1} = x$, bu yerda p – haqiqiy son
8. $2\left(\sqrt{2-\sqrt{3}}\right)^x + \left(4\sqrt{2+\sqrt{3}}\right)^x = 3 \cdot 2^x$