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O‘ZGARISH CHIZIG‘IGA EGA PARABOLIK-GIPERBOLIK TIPDAGI TENGLAMA UCHUN INTEGRAL ULASH SHARTLI CHEGARAVIY MASALA

Annotatsiya: Ushbu ishda Riman-Liuvill kasr tartibli hosila ishtrok etgan aralash tenglama uchun aralash sohada umumiy integral shartli chegaraviy masalaning bir qiymatli yechilishi tadqiq qilinadi.

Kalit so‘zlar: Aralash tenglama, kasr tartibli hosila, integral ulash sharti.

ИНТЕГРАЛЬНАЯ СВЯЗЬ УСЛОВНАЯ ГРАНИЧНАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЯ ПАРАБОЛО- ГИПЕРБОЛИЧЕСКОГО ТИПА С ПЕРЕМЕННОЙ ПРЯМОЙ

Аннотация: В работе исследуется однозначное решение общей интегральной условно-краевой задачи в смешанном поле для смешанного уравнения, включающего дробную производную Римана-Лиувилля.

Ключевые слова: Смешанное уравнение, дробная производная, интегральное условие связности.

EDGE PROBLEM FOR INTEGRAL CONNECTION IN CONDITIONAL BOUNDARY FOR A PARABOLIC-HYPERBOLIC TYPE EQUATION WITH A VARIABLE LINE

Abstract: In this work a unique solvability of a boundary problem with general integral gluing condition for mixed equation involving the Riemann-Liouville fractional derivative has been proved.

Keywords: Mixed equation, fractional derivative, integral gluing condition.

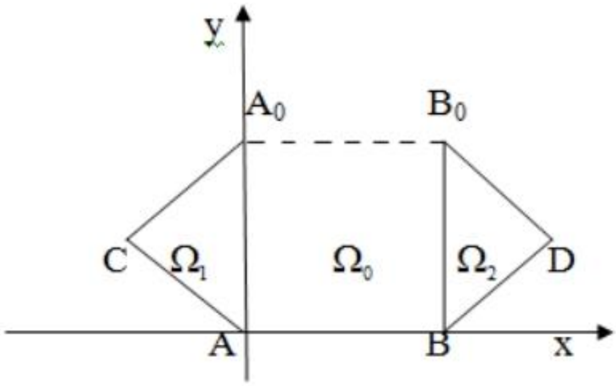
$$f(x,y)=\begin{cases} U_{xx}(x,y)-D_{0y}^{\alpha}U(x,y), & (x,y)\in\Omega_0, \\ U_{xx}(x,y)-U_{yy}(x,y), & (x,y)\in\Omega_i \quad (i=1,2) \end{cases} \tag{1}$$

tenglamani $\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup AA_0\cup BB_0$ aralash sohada tadqiq qilamiz.

Bu yerda $f(x,y)$ -berilgan funksiya, $D_{0y}^{\alpha}U$ esa α kasr tartibli Riman-Liuvill integro-differensial operatori bo‘lib, u $0<\alpha<1$ uchun quydagicha aniqlangan $[1]$:

$$D_{0y}^{\alpha} g(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-z)^{-\alpha} g(z) dz.$$

(1) tenglama uchun Ω sohada



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quydagi masalani tadqiq etamiz:

1-Masala. (1) tenglamaning Ω sohada

$$U(x,y) \in C(\bar{\Omega}) \cap AC^1(\Omega_0) \cap C^2(\Omega_i), \quad U_{xx} \in C(\Omega_0)$$

funksiyalar sinfiga tegishli quyidagi shartlarni qanoatlantiradigan regulyar yechimi topilsin:

$$U(x,0) = 0, \qquad 0 \le x \le 1, \tag{2}$$

$$U|_{A_0C} = \varphi(y), \qquad \frac{1}{2} \le y \le 1, \tag{3}$$

$$U|_{B_0D} = \psi(y), \qquad \frac{1}{2} \le y \le 1, \tag{4}$$

$$U_x(0+,y) = I_1(U(x,y)|_{x=0-}), \quad U_y(0+,y) = U_y(0-,y), \qquad 0 < y < 1, \tag{5}$$

$$U_x(1-0,y) = I_2(U_x(x,y)|_{x=1+0}), \quad U_y(1-0,y) = U_y(1+0,y), \quad 0 < y < 1. \tag{6}$$

Bu yerda $\varphi(y)$, $\psi(y)$-berilgan funksiyalar, I_1, I_2 lar esa hozircha ixtiyoriy integral operatorlar.

Bunday tipdagi masalalar I_1 *va* I_2 integral operatorlarning maxsus ko'rinishida [2] da ($\alpha = 1$ holda) hamda $0 < \alpha < 1$ uchun [3] tadqiq etilgan.

(1) tenglamaning Ω_0 sohada (2) va

$$U_x(0+,y) = \nu_0^+(y), \qquad U_x(1-0,y) = \nu_1^-(y), \qquad 0 < y < 1 \tag{7}$$

shartlarni qanoatlantiruvchi yechimi quydagicha yoziladi [4] :

$$U(x,y) = \int_0^y \nu_1^-(\eta) G(x,y;1,\eta)\,d\eta - \int_0^y \nu_0^+(\eta) G(x,y;0,\eta)\,d\eta - $$
$$-\int_0^y \int_0^1 f(\xi,\eta) G(x,y;\xi,\eta)\,d\xi\,d\eta, \tag{8}$$

bu yerda

$$G(x,y;\xi,\eta) = \frac{(y-\eta)^{\beta-1}}{2} \sum_{n=-\infty}^{+\infty} \left[e_{1,\beta}^{1,\beta}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\beta}} \right) + e_{1,\beta}^{1,\beta}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\beta}} \right) \right], \tag{9}$$

$e_{1,\beta}^{1,\beta}(z) = \sum_{n=0}^{+\infty} \frac{z^n}{n!\,\Gamma(\beta-\beta n)}$ $\rightarrow$ Rayt tipidagi funksiya [4], $\beta = \frac{\alpha}{2}$

(1) tenglama uchun Ω_1 va Ω_2 sohalardagi Koshi masalasi yechimini Dalamber formulasi orqali yozib olamiz [5]:

$$U(x,y) = \frac{1}{2}\left\{ \tau_0^-(y+x) + \tau_0^-(y-x) + \int_{y-x}^{y+x} \nu_0^-(t)\,dt + \int_0^y \int_{x-y+\eta}^{x+y-\eta} f(\xi,\eta)\,d\xi\,d\eta \right\},$$
$$(x,y) \in \Omega_1 \tag{10}$$

$$U(x,y) = \frac{1}{2}\left\{ \tau_1^+(y-x+1) + \tau_1^+(y+x-1) + \int_{y+x-1}^{y-x+1} \nu_1^+(t)\,dt + \int_0^y \int_{1-x-y+\eta}^{1-x+y-\eta} f(\xi,\eta)\,d\xi\,d\eta \right\},$$
$$(x,y) \in \Omega_2 \tag{11}$$

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Bu yerda $U(-0,y)=\tau_0^-(y)$, $U(1+0,y)=\tau_1^+(y)$.

(10) ni (3) ga qo‘yamiz:

$$U(y-1,y)=\varphi(y)=\frac{1}{2}\left\{\tau_0^-(y+y-1)+\tau_0^-(y-y+1)+\int_{y-y+1}^{y+y-1}\nu_0^-(t)dt+\int_0^y\int_{y-1-y+\eta}^{y-1+y-\eta}f(\xi,\eta)d\xi d\eta\right\},$$

$$\frac{1}{2}\le y\le 1.$$

$$\varphi(y)=\frac{1}{2}\left\{\tau_0^-(2y-1)+\tau_0^-(1)+\int_1^{2y-1}\nu_0^-(t)dt+\int_0^y\int_{\eta-1}^{2y-\eta-1}f(\xi,\eta)d\xi d\eta\right\},$$

yoki (2) ni hisobga olsak $\tau_0^-(1)=0$ va y bo‘yicha bir marta differensiallab y ni $\frac{y+1}{2}$ ga almashtirib

$$\varphi'\left(\frac{y+1}{2}\right)=\tau_0^{-\prime}(y)+\nu_0^-(y)+\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\,,\qquad 0<y<1 \tag{12}$$

ni olamiz.

Xuddi shuningdek, (11) ni (4) ga qo‘yib, yuqoridagidek amallarni bajarganda quyidagini olamiz:

$$\psi'\left(\frac{y+1}{2}\right)=\tau_1^{+\prime}(y)+\nu_1^+(y)+\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\,,\qquad 0<y<1. \tag{13}$$

(5) va (6) ulash shartlarini quydagicha yozib olish mumkin:

$$\nu_0^+(y)=I_1(\nu_0^-(y)),\qquad \nu_1^-(y)=I_2(\nu_1^+(y)),\qquad 0<y<1, \tag{14}$$

(12), (13) ni hisobga olsak, (14) dan

$$\nu_0^+(y)=I_1\left[\varphi'\left(\frac{y+1}{2}\right)-\tau_0^{-\prime}(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right],$$

$$\nu_1^-(y)=I_2\left[\psi'(\frac{y+1}{2})-\tau_1^{+}{}'(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right] \tag{15}$$

ifodalarni hosil qilamiz. (15) ni (8) ga qo‘yamiz:

$$U(x,y)=\int_0^y I_2\left[\psi'(\frac{y+1}{2})-\tau_1^{+}{}'(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right]G(x,y;1,\eta)d\eta-$$
$$-\int_0^y I_1\left[\varphi'(\frac{y+1}{2})-\tau_0^{-}{}'(y)-\int_0^{\frac{y+1}{2}}f(y-\eta,\eta)d\eta\right]G(x,y;0,\eta)d\eta-$$
$$-\int_0^y\int_0^1 f(\xi,\eta)G(x,y;\xi,\eta)d\xi d\eta \tag{16}$$

(16) da $x\to 0+\ va\ x\to 1-0$ holatlarda limitlarga o‘tamiz:

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)I_2\left[\psi'(\frac{\eta+1}{2})-\tau_1^{+}{}'(\eta)-\int_0^{\frac{\eta+1}{2}}f(\eta-\xi,\xi)d\xi\right]d\eta-$$
$$-\int_0^y G(0,y;0,\eta)I_1\left[\varphi'(\frac{\eta+1}{2})-\tau_0^{-}{}'(\eta)-\int_0^{\frac{\eta+1}{2}}f(\eta-\xi,\xi)d\xi\right]d\eta-$$
$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta\,, \tag{17}$$

$$\tau_1^+(y)=\int_0^y G(1,y;1,\eta)I_2\left[\psi'(\frac{\eta+1}{2})-\tau_1^{+}{}'(\eta)-\int_0^{\frac{\eta+1}{2}}f(\eta-\xi,\xi)d\xi\right]d\eta-$$
$$-\int_0^y G(1,y;0,\eta)I_1\left[\varphi'(\frac{\eta+1}{2})-\tau_0^{-}{}'(\eta)-\int_0^{\frac{\eta+1}{2}}f(\eta-\xi,\xi)d\xi\right]d\eta-$$
$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)d\xi d\eta\,. \tag{18}$$

Agar I_1 va I_2 integral operatorlarni quyidagi ko‘rinishda olsak:

$$I_1(g)=\alpha_1 g(\eta)+\int_0^{\eta} g(z)K_1(\eta,z)dz,$$

$$I_2(g)=\alpha_2 g(\eta)+\int_0^{\eta} g(z)K_2(\eta,z)dz, \tag{19}$$

(17) va (18) larni Volterra integral tenglamalar sistemasiga keltirsak bo‘ladi:

$$\tau_0^-(y)=\int_0^y G(0,y;1,\eta)\left(\begin{array}{l}\alpha_2\left(\left(\psi'\dfrac{\eta+1}{2}\right)-\tau_1^{+\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\right)+\\ +\displaystyle\int_0^{\eta}K_2(\eta,z)dz\left[\psi'\left(\dfrac{z+1}{2}\right)-\tau_1^{+\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\left(\begin{array}{l}\left(\alpha_1\left(\left(\varphi'\dfrac{\eta+1}{2}\right)-\tau_0^{-\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\right)+\right.\\ +\displaystyle\int_0^{\eta}K_1(\eta,z)dz\left[\varphi'\left(\dfrac{z+1}{2}\right)-\tau_0^{-\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta, \tag{20}$$

$$\tau_1^+(y)=\int_0^y G(1,y;1,\eta)\left(\begin{array}{l}\alpha_2\left(\psi'\left(\dfrac{\eta+1}{2}\right)-\tau_1^{+\prime}(\eta)-\displaystyle\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right)+\\ +\displaystyle\int_0^{\eta}K_2(\eta,z)dz\left[\psi'\left(\dfrac{z+1}{2}\right)-\tau_1^{+\prime}(z)-\displaystyle\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y G(1,y;0,\eta)\left(\begin{array}{l}\alpha_1\left(\varphi'\left(\frac{\eta+1}{2}\right)-\tau_0^{-\prime}(\eta)-\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)+\\ +\int_0^{\eta} K_1(\eta,z)\,dz\left[\varphi'\left(\frac{z+1}{2}\right)-\tau_0^{-\prime}(z)-\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)\,d\xi\right]\end{array}\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(1,y;\xi,\eta)\,d\xi\,d\eta\,. \tag{21}$$

Endi (20) ni soddalashtiramiz:

$$\tau_0^-(y)=\alpha_2\int_0^y G(0,y;1,\eta)\psi'\left(\frac{\eta+1}{2}\right)d\eta-\alpha_2\int_0^y G(0,y;1,\eta)\tau_1^{+\prime}(\eta)\,d\eta-$$

$$-\alpha_2\int_0^y G(0,y;1,\eta)\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)d\eta+\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\psi'\left(\frac{z+1}{2}\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\tau_1^{+\prime}(z)\,dz\right)d\eta-\int_0^y G(0,y;1,\eta)\left(\int_0^{\eta} K_2(\eta,z)\left(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)\,d\xi\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\alpha_1\varphi'\left(\frac{\eta+1}{2}\right)d\eta+\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)\,d\eta+$$

$$+\int_0^y G(0,y;0,\eta)\alpha_1\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)\,d\xi\right)d\eta-\int_0^y G(0,y;0,\eta)\left(\int_0^{\eta} K_1(\eta,z)\varphi'\left(\frac{z+1}{2}\right)dz\right)d\eta+$$

$$+\int_0^y G(0,y;0,\eta)\left(\int_0^{\eta} K_1(\eta,z)\tau_0^{-\prime}(z)\,dz\right)d\eta+\int_0^y G(0,y;0,\eta)\left(\int_0^{\eta} K_1(\eta,z)\left(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)\,d\xi\right)dz\right)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)\,d\xi\,d\eta. \tag{22}$$

(22) da $\tilde{K}_1(y,z)$ va $\tilde{F}(y)$ belgilashlar kiritsak u quyidagi ko‘rinishga keladi:

$$\tau_0^-(y)-\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)d\eta-\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)\tau_0^{-\prime}(z)dz)d\eta=\tilde{F}(y)$$

$$\tau_0^-(y)-\int_0^y G(0,y;0,\eta)\alpha_1\tau_0^{-\prime}(\eta)d\eta-\int_0^y \tau_0^{-\prime}(z)K_1(\eta,z)dz=\tilde{F}(y)$$

yoki

$$\tau_0^-(y)-\int_0^y \tau_0^{-\prime}(\eta)\left[\alpha_1 G(0,y;0,\eta)+\tilde{K}_1(y,\eta)\right]d\eta=\tilde{F}(y), \tag{23}$$

bu yerda

$$\tilde{K}_1(y,z)=\int_z^y G(0,y;0,\eta)K_1(\eta,z)d\eta,$$

$$\tilde{F}(y)=\int_0^y G(0,y;1,\eta)\alpha_2\psi'\left(\frac{\eta+1}{2}\right)d\eta-\int_0^y G(0,y;1,\eta)\alpha_2\tau_1^{+\prime}(\eta)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)\alpha_2\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right)d\eta+\int_0^y G(0,y;1,\eta)\left(\int_0^\eta K_2(\eta,z)\psi'\left(\frac{z+1}{2}\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;1,\eta)(\int_0^\eta K_2(\eta,z)\tau_1^{+\prime}(z)dz)d\eta-\int_0^y G(0,y;1,\eta)\left(\int_0^\eta K_2(\eta,z)\left(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi\right)dz\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)\alpha_1\varphi'\left(\frac{\eta+1}{2}\right)d\eta+\int_0^y G(0,y;0,\eta)\alpha_1\left(\int_0^{\frac{\eta+1}{2}} f(\eta-\xi,\xi)d\xi\right)d\eta-$$

$$-\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)\varphi'\left(\frac{z+1}{2}\right)dz)d\eta+\int_0^y G(0,y;0,\eta)(\int_0^\eta K_1(\eta,z)(\int_0^{\frac{z+1}{2}} f(z-\xi,\xi)d\xi)dz)d\eta-$$

$$-\int_0^y\int_0^1 f(\xi,\eta)G(0,y;\xi,\eta)d\xi d\eta.$$

(23) ni (9) dan foydalanib quyidagicha yozib olamiz:

$$\tau_0^-(y)-\frac{\alpha_1}{\Gamma(\beta)}\int_0^y\frac{\tau_0^{-\prime}(z)}{(y-z)^{1-\beta}}dz-\alpha_1\int_0^y\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}(y-z)^{\beta-1}e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(y-z)^{\beta}}\right)dz-$$
$$-\int_0^y\tau_0^{-\prime}(z)\tilde{K}_1(y,z)dz=\tilde{F}(y) \quad (24)$$

yoki

$$\frac{1}{\Gamma(\beta)}\int_0^y\frac{\tau_0^{-\prime}(z)}{(y-z)^{1-\beta}}dz=\frac{1}{\alpha_1}\left[\tau_0^-(y)-\alpha_1\int_0^y\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(y-z)^{\beta}}\right)}{2(y-z)^{1-\beta}}dz-\int_0^y\tau_0^{-\prime}(z)\tilde{K}_1(y,z)dz-\tilde{F}(y)\right] \quad (25)$$

Riman-Liuvill integrali ko‘rinishdan foydalansak,

$$I_{0y}^{\beta}\left(\tau_0^{-\prime}(y)=\tilde{\tilde{F}}(y)\right) \quad (26)$$

hosil bo‘ladi. (26) ning har ikki tomoniga ${}_cD_{0y}^{\beta}$ operatorni ta’sir ettiramiz:

$${}_cD_{0y}^{\beta}\left(I_{0y}^{\beta}\tau_0^{-\prime}(y)\right)={}_cD_{0y}^{\beta}\left(\tilde{\tilde{F}}(y)\right). \quad (27)$$

(28) ning o‘ng tomonini hisoblab olamiz:

$${}_cD_{0y}^{\beta}\left(\tau_0^-(y)\right)=\frac{1}{\Gamma(\beta)}\int_0^y(y-z)^{-\beta}\tau_0^{-\prime}(z)dz, \quad (28)$$

$${}_cD_{0y}^{\beta}\left[\int_0^y\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(y-z)^{\beta}}\right)}{(y-z)^{1-\beta}}dz\right]=\frac{1}{\Gamma(1-\beta)}\int_0^y(y-s)^{-\beta}\frac{d}{ds}\left(\int_0^s\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}dz\right)ds=$$
$$=\frac{1}{\Gamma(1-\beta)}\int_0^y(y-s)^{-\beta}ds\left[\lim_{s\to z}\left(\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}dz\right)+\int_0^s\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\n\neq0}}^{+\infty}\frac{\partial}{\partial s}\left(\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right)dz\right], \quad (29)$$

bu yerda

$$\lim_{s\to z}\left(\tau_0^{-\prime}(z)\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right)=0$$

Bu tasdiqning isbotini 1-lemmada keltirib o‘tamiz. Endi

$$\frac{\partial}{\partial s}\left(\frac{e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)}{(s-z)^{1-\beta}}\right)$$ ni hisoblab olamiz:

$$\frac{\partial}{\partial s}\left((s-z)^{\beta-1}e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(s-z)^{\beta}}\right)\right)=(s-z)^{\beta-2}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)$$

Olingan natijalarni (29) ga qo‘ysak,

$$\frac{1}{\Gamma(1-\beta)}\int_0^y (y-s)^{-\beta}\left(\int_0^s \tau_0^{-\prime}(z)(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)dz\right)ds=$$

$$=\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(z)\,dz\int_0^y (y-s)^{-\beta}(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)ds \qquad (30)$$

ko‘rinishga keladi.(30) da $M(y,z)$ belgilash kiritib olamiz:

$$M(y,z)=\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{+\infty}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)ds$$

$M(y,z)$ ni soddalashtirishni 2-lemmada ko‘ramiz:

(27) ning o‘ng tomonidagi oxirgi ifodani hisoblaymiz:

$$
{}_cD_{0y}^{\beta}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(y,z)dz\right]=
$$

$$
=\frac{1}{\Gamma(1-\beta)}\int_0^y (y-t)^{-\beta}\frac{\partial}{\partial t}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(t,z)dz\right]dt=
$$

$$
=\frac{1}{\Gamma(1-\beta)}\int_0^y (y-t)^{-\beta}\left[\int_0^y \tau_0^{-\prime}(z)\tilde{K}_1(t,t)+\int_0^t \tau_0^{-\prime}(z)\frac{\partial\tilde{K}_1(t,z)}{\partial t}dz\right]dt=
$$

$$
=\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(t)\frac{\tilde{K}_1(t,t)}{(y-t)^{\beta}}dt+\frac{1}{\Gamma(1-\beta)}\int_0^y \frac{dt}{(y-t)^{\beta}}\cdot
$$

$$
\cdot\int_0^t \tau_0^{-\prime}(z)\frac{\partial\tilde{K}_1(t,z)}{\partial t}dz=\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(t)\frac{\tilde{K}_1(t,t)}{(y-t)^{\beta}}dt+
$$

$$
+\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(z)dz\int_0^y \frac{\frac{\partial\tilde{K}_1(t,z)}{\partial t}}{(y-t)^{\beta}}dt=
$$

$$
=\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(s)\left[\frac{\tilde{K}_1(s,s)}{(y-s)^{\beta}}+\int_0^y \frac{\frac{\partial\tilde{K}_1(t,s)}{\partial t}}{(y-t)^{\beta}}dt\right]ds.
$$

Demak,

$$
\tau_0^{-\prime}(y)-\int_0^y \tau_0^{-\prime}(z)\left[\begin{array}{l}\dfrac{(y-z)^{-\beta}}{\alpha_1\Gamma(1-\beta)}+\dfrac{M(y,z)}{\Gamma(1-\beta)}+\\ +\dfrac{1}{\Gamma(1-\beta)}\left(\dfrac{\tilde{K}_1(z,z)}{(y-z)^{\beta}}+\displaystyle\int_0^y \frac{\frac{\partial\tilde{K}_1(t,s)}{\partial t}}{(y-t)^{\beta}}dt\right)\end{array}\right]dz. \quad (31)
$$

(31) da $\bar{F}(y)$ *va* $\bar{K}(y,z)$ belgilashlar kiritib olsak,

$$
\tau_0^{-\prime}(y)-\frac{1}{\Gamma(1-\beta)}\int_0^y \tau_0^{-\prime}(z)\bar{K}(y,z)=\bar{F}(y) \quad (32)
$$

ni olamiz, bu yerda

$$\bar{K}(y,z)=\frac{1}{\alpha_1}\frac{\Gamma(1-\beta)}{\Gamma(\beta)}\frac{1}{(y-z)^{\beta}}+M(y,z)+\frac{\tilde{K}_1(z,z)}{(y-z)^{\beta}}+\int_0^y\frac{\frac{\partial\tilde{K}_1(t,z)}{\partial t}}{(y-t)^{\beta}}dt$$

$$\bar{F}(y)=-\frac{1}{\alpha_1}\,{}_cD_{0y}^{\beta}\tilde{F}(y)$$

$\bar{F}(y)$ - uzuluksiz, $\bar{K}(y,z)$ - kichik maxsuslikka ega.

1-lemma.

Agar $\frac{1}{3}\le\beta\le\frac{1}{2}$ va $n\neq 0$ bo‘lsa, u holda

$$\lim_{s\to z}(z-s)^{\beta-1}\,e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)=0$$

bo‘ladi.

Isbot:

Avvalo,

$$e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)=\underbrace{\frac{-2n}{(z-s)^{\beta}}e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)}\frac{(z-s)^{\beta}}{-2n}$$

tarizda shakl almashtiramiz va

$$ze_{\alpha,\beta}^{\mu,\delta}(z)=e_{\alpha,\beta}^{\mu-\alpha,\delta+\beta}(z)-\frac{1}{\Gamma(\mu-\alpha)\Gamma(\delta+\beta)}$$

$formulada\,\mu=1,\alpha=1,\quad \delta=\beta,\ \beta=\beta$ holda

$$e_{1,\beta}^{1,\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)=\left(\frac{(z-s)^{\beta}}{-2n}\right)^2\left(\frac{-2n}{(z-s)^{\beta}}e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right)$$

ni olamiz [4].

Demak,

$$\lim_{s\to z}\frac{(z-s)^{3\beta-1}}{(-2n)^2}\left(\frac{-2n}{(z-s)^{\beta}}e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right)$$

$$Agar\ 3\beta-1\geq 0\ \ yoki\ \ \beta\geq\frac{1}{3}\ \ desak$$

$$\lim_{s\to\infty} z e_{\alpha,\beta}^{\mu,\delta}(z)=-\frac{1}{\Gamma(\mu-\alpha)\Gamma(\delta+\beta)}\ \ ni\ hisobga\ olib$$

$$\lim_{s\to z}\frac{(z-s)^{3\beta-1}}{(-2n)^2}\left(\frac{-2n}{(z-s)^{\beta}}e_{1,\beta}^{0,2\beta}\left(\frac{-2n}{(z-s)^{\beta}}\right)\right)=\frac{0}{(-2n)^2}\frac{-1}{\Gamma(0-1)\Gamma(3\beta)}=0$$

ni hosil qilamiz[4]. $\beta\in\left[0,\frac{1}{2}\right]$ ekanligini hisobga olsak,demak $\beta\in\left[\frac{1}{3},\frac{1}{2}\right]$ bo'lishi kerak.

2-lemma. $n\neq 0$ uchun

$$\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2}e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(z-s)^{\beta}}\right)ds=0$$

tenglik o'rinli.

Isbot:

$$e_{\alpha,\beta}^{\mu,\delta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\mu)\Gamma(\delta-\beta k)}\ \ formulaga\ ko'ra$$

$$\mu=1,\ \alpha=1,\quad \delta=\beta-1,\ \beta=\beta$$

holda quyidagini olamiz[4]:

$$e_{1,\beta}^{1,\beta-1}\left(\frac{-2n}{(s-z)^{\beta}}\right)=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k(s-z)^{-\beta k}}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}.$$

Demak,

$$M(y,z)=\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2}\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k(s-z)^{-\beta k}}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}ds =$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\int_z^y (y-s)^{-\beta}(s-z)^{\beta-2-\beta k}\,ds.$$

Bu yerda

$$(y-z)\xi+z=s,\qquad s-z=(y-z)\xi$$
$$y-s=y-(y-z)\xi+z=(1-\xi)(y-z)$$
$$(y-z)d\xi=ds$$

almashtirish bajarsak,

$$M(y,z)=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\int_0^1((1-\xi)(y-z))^{-\beta}((y-z)\xi)^{\beta-2-\beta k}(y-z)d\xi=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\int_0^1(1-\xi)^{-\beta}\xi^{\beta-2-\beta k}(y-z)^{-1-\beta k}\,d\xi=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\frac{1}{(y-z)^{1+\beta k}}\int_0^1(1-\xi)^{-\beta}\xi^{\beta-2-\beta k}\,d\xi$$

hosil bo‘ladi.

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}\,dx=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\ ni\ hisobga\ olib,$$

Endi $a=\beta-\beta k-1,\quad b=1-\beta\ da$

$$M(y,z)=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\frac{1}{(y-z)^{1+\beta k}}\frac{\Gamma(\beta-\beta k-1)\Gamma(1-\beta)}{\Gamma(\beta-\beta k-1+1-\beta)}=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k}{\Gamma(k+1)\Gamma(\beta-1-\beta k)}\frac{1}{(y-z)^{1+\beta k}}\frac{\Gamma(\beta-1-\beta k)\Gamma(1-\beta)}{\Gamma(-\beta k)}=$$

$$=\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k\Gamma(1-\beta)}{\Gamma(k+1)\Gamma(-\beta k)(y-z)^{1+\beta k}}$$

$$\sum_{k=0}^{+\infty}\frac{(-1)^k(2n)^k\Gamma(1-\beta)}{\Gamma(k+1)\Gamma(-\beta k)(y-z)^{1+\beta k}}=\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)$$

$$M(y,z)=\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)$$ ni hosil qilamiz.

3-lemma. Agar $\beta\in\left[\frac{1}{3},\frac{1}{2}\right]$ va $n\neq 0$ bo‘lsa, u holda $\lim\limits_{z\to y}\frac{1}{y-z}e_{1,\beta}^{1,0}\left(\frac{-2n}{(y-z)^{\beta}}\right)=0$.

Bu lemmaning isboti 1-lemmaning isbotiga o‘xshash bo‘lib, $\lim\limits_{a\to -2}\frac{1}{\Gamma(a)}=0$ tasdiqdan foydalaniladi.

Bunda ham $\beta\in\left[\frac{1}{3},\frac{1}{2}\right]$ bo‘lishi talab etiladi.

Demak, $\tau_1^{+}(y)$ funksiyani vaqtincha ma’lum desak, (32) ni 2-tur Volterra integral tenglamasi sifatida qarab, uning yechimini rezolventa orqali yozib olish mumkin:

Bu yechimni (21) ga olib borib qo‘yilganda $\tau_1^{+}(y)$ ga nisbatan yana bir 2-tur Volterra integral tenglamasini olamiz.

Berilganlarga ma’lum shartlar asosida bu integral tenglamaning yechimi ham rezolventa orqali yoziladi. (15) munosabatlardan va (5), (6) ulash shartlari asosida $\nu_0^{\pm}(y),\ \nu_1^{\pm}(y)$ funksiyalar topiladi. Undan so‘ng Ω_0 sohada yechim (8) formula, Ω_1 sohada (10), Ω_2 sohada esa (11) formulalar bilan tiklanadi.

Masala ekvivalaent tarzda 2-tur Volterra integral tenglamalar sistemasiga keltirilganligi tufayli, masala yechimi yagona bo‘ladi.

Demak, quyidagi tasdiq o‘rinli:

Teorema. Agar $\alpha\in\left[\frac{2}{3},1\right]$, $\varphi,\psi\in C^1[0,1]\cap C^2(0,1)$, $f(x,y)\in(\bar{\Omega})\cap C^1(\Omega)$,

$I_1\ va\ I_2$ integral operatorlar (19) ko‘rinishda berilgan bo‘lsa va $\alpha_1\neq 0,\ \alpha_2\neq 0$ bo‘lganda 1-masalaning yechimi mavjud va yagona bo‘ladi.

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