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TAKRORIY SINOVLAR. BERNULLI FO'RMULASINING MASALALARGA TADBIQLARI VA QULAY YECHIMLARI

Annotatsiya: Ushbu maqolada Ehtimollar nazariyasi va matematik statistika fanining Takroriy sinovlar, Bernulli fo'rmulasi mavzusi sodda tushuntirib o'tilgan. Mavzuga doir yangi masalalar keltirilgan va uning qulay usullardagi yechimlari tushuntirib ko'rsatilgan.

Kalit so'zlar: Erkli sinov, Hodisalar, ro'y berish ehtimoli, n ta erkli sinov, ro'y berish ehtimoli $p(0<p<1)$

ПОВТОРНЫЕ ИСПЫТАНИЯ. ПРИЛОЖЕНИЯ ФОРМУЛЫ БЕРНУЛИ К ЗАДАЧАМ И УДОБНЫЕ РЕШЕНИЯ

Аннотация: В данной статье простым языком объясняется тема повторных испытаний, формулы Бернулли теории вероятностей и математической статистики. Представлены новые вопросы, связанные с темой, и в удобной форме объяснены их решения.

Ключевые слова: Случайный тест, События, вероятность возникновения, случайный тест, вероятность возникновения.

REPEAT TESTS. APPLICATIONS OF BERNOULLI'S FORMULA TO PROBLEMS AND CONVENIENT SOLUTIONS

Annotation: In this article, the topic of repeated trials, Bernoulli's formula of probability theory and mathematical statistics is explained in a simple way. New issues related to the topic are presented and their solutions are explained in convenient ways.

Keywords: Random test, Events, probability of occurrence, random test, probability of occurrence.

Asosiy qism: O'tkazilayotgan bir necha sinovlarning har birida qandaydir A hodisaning ro'y berish ehtimoli qolgan sinovlarning natijalariga bog'liq bo'lmasa, u holda bunday sinovlar A hodisaga nisbatan erkli deb ataladi.

Birgalikda bo'lmagan xodisalar ehtimollarini qo'shish teoremasi: Ikkita birgalikda bo'lmagan hodisadan istalgan birining ro'y berish ehtimoli bu xodisalar ehtimollarining yig'indisiga teng:

$$\mathrm{P}(\mathrm{A}+\mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B}).$$

Natija: Xar bir ikkitasi birgalikda bo'lmagan bir nechta xodisalardan istalgan birining ro'y berish ehtimoli bu xodisalar ehtimollarining yig'indisiga teng:

$$P(A_1 + A_2 + \ldots + A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

Birgalikda bo‘lgan xodisalar ehtimollarini qo‘shish teoremasi: Ikkita birgalikda bo‘lgan hodisadan kamida bittasining ro‘y berish ehtimoli bu hodisalarning ehtimollari yig‘indisidan ularning birgalikda ro‘y berish ehtimolini ayrilganiga teng.

$$P(A+B) = P(A) + P(B) - P(AB).$$

Teorema istalgan chekli sondagi birgalikda bo‘lgan hodisalar uchun umumlashtirilishi mumkun. Masalan, uchta birgalikda bo‘lgan xodisa uchun:

$$P(A+B+C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

Erkli xodisalar ehtimollarini ko‘paytirish teoremasi: Ikkita erkli xodisaning birgalikda ro‘y berish ehtimoli bu xodisalar ehtimollarini ko‘paytmasiga teng:

$$P(AB) = P(A)\ P(B).$$

Natija: Bir nechta bog‘liq hodisalarning birgalikda ro‘y berish ehtimoli bu hodisalar ehtimollariniko‘paytirilganiga teng:

$$P(A_1 A_{2\ldots} A_n) = P(A_1)\ P(A_2)\ \ldots\ P(A_n).$$

Bog‘liq hodisalar ehtimollarini ko‘paytirish teoremasi: Ikkita bog‘liq hodisaning birgalikda ro‘y berish ehtimoli ulardan birining ehtimolini ikkinchisining shartli extimoliga ko‘paytirilganiga teng:

$$P(AB) = P(A)\ P_A(B),$$

$$P(AB) = P(B)\ P_B(A).$$

Natija: Bir nechta bog‘liq hodisalarning birgalikda ro‘y berish ehtimoli ulardan birining ehtimolini qolganlarining shartli ehtimollariga ko‘paytirilganiga teng, shu bilan birga, xar bir keyingi hodisaning ehtimolini oldingi hamma hodisalar ro‘y berdi degan farazda hisoblaydi:

$P(A_1 A_2 A_{3\ldots} A_n) = P(A_1)\ P_{A_1}(A_2)\ P_{A_1 A_2}(A_3)\ \ldots\ P_{A_1 A_2 \ldots A_{n-1}}(A_n)$ bu yerda $P_{A_1 A_2 \ldots A_{n-1}} - (A_n)$ xodisaning $A_{1,} A_{2,\ldots,} A_{n-1}$ xodisalar ro‘y berdi degan farzda xisoblangan ehtimoli.

1-masala: Kutubxona stellajida tasodifiy tartibda 15 ta darslik terib qo‘yilgan bo‘lib, ulardan 5 tasi muqovalidur. Kutubxonachi ayol tavakkaliga 3 ta darslik oldi. Olingan darslikning hech bo‘lmaganida bittasi muqovali bo‘lish (A xodisa) ehtimolini toping.

Yechish: Birinchi usul. Olingan uchta darslik xech bo‘lmaganda bittasi muqovali bo‘lish talabi quyidagi uchta birgalikda bo‘lmagan xodisadan istalgan biri ro‘y berganda bajariladi: B-bitta darslik muqovali, ikkitasi muqovasiz, C- ikkita darslik muqovali, bittasi muqovasiz, D- uchala darslik muqovali.

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Bizni qiziqtirayotgan A xodisani (olingan uchta darslikning hech bo‘lmaganda bittasi muqovali bo‘lishi) bu hodisaning yig‘indisi ko‘rinishida ifodalash mumkun:

A=B+C+D.

Qo‘shish teoremasiga ko‘ra:

$$\mathrm{P}(\mathrm{A}) = \mathrm{P}(\mathrm{B}) + \mathrm{P}(C) + \mathrm{P}(D) \qquad (1)$$

B, C va D xodisalarning ehtimollarini topamiz

$$\mathrm{P}(\mathrm{B}) = \frac{C\frac{1}{5}\ C\frac{2}{10}}{C\frac{3}{15}} = \frac{45}{91}, \qquad \mathrm{P}(C) = \frac{C\frac{2}{5}\ C\frac{1}{10}}{C\frac{3}{15}} = \frac{20}{91}, \qquad \mathrm{P}(D) = \frac{C\frac{3}{5}}{C\frac{3}{15}} = \frac{2}{91}.$$

Bu extimollarni (1) tenglikka qo‘yib , quyidagini hosil qilamiz:

P (A)=45/91+2091+2/91=67/91.

Ikkinchi usul: A xodisa (olingan uchta darslikdan xech bo‘lmaganda bittasi muqovali) va A xodisa (olingan darslikning bittasi ham muqovali emas) qarama-qarshi xodisalardir, shuning uchun

P(A)+P(A)=1

(qarama-qarshi xodisalarning ehtimollari yig‘indisi birga teng).

Bunda

P(A)=1-P(A).

A xodisaning (olingan darsliklarning bittasi ham muqovali emas) ro‘y berish ehtimoli

$$\mathrm{P}(\mathrm{A}) = \frac{C\frac{3}{10}}{C\frac{3}{15}} = \frac{24}{91}.$$

Izlanyotgan ehtimol:

P(A)=1-P(A)=1-24/91=67/91.

Bernulli formulasi: Har birida xodisaning ro‘y berish ehtimoli $p\left(0 < p < 1\right)$ ga teng bo‘lgan n ta erkli sinovda hodisaning (qaysi tartibda bo‘lishidan qat’iy nazar) rosa k martasida ro‘y berish ehtimoli

$$P_n\left(k\right) = C_n^k p^k q^{n-k} \qquad (1)$$

formula bilan hisoblanadi. Ushbu (1) formuladan quyidagi formulani ham keltirib chiqarish mumkin. $P_n(k)=\frac{n!}{k!(n-k)!}p^k q^{n-k}$ bu yerda $q=1-p.$ ga teng.

2-masala. Tangani 4 marta tashlaganda 2 martasida gerb tomon tushishi ehtimolini toping.

Yechish: Tangani bir marta tashlaganda gerb tomon tushishi ehtimoli $p=\frac{1}{2}$ ga teng, demak bundan $q=1-p=1-\frac{1}{2}=\frac{1}{2}$ ga teng. Masalaning shartiga ko‘ra $n=4$ va $k=2$ ga teng. Ushbu qiymatlarni (1) formulaga qo‘yilsa $P_4(2)=\frac{4!}{2!(4-2)!}\ (\frac{1}{2})^2\ (\frac{1}{2})^{4-2}=6\ \frac{1}{4}\ \frac{1}{4}=\frac{3}{8}$ ko‘rinishidagi qiymatga erishamiz.

Javob: $\frac{3}{8}$

Hodisalarning 1) k dan kam marta; 2) k dan ko‘p marta; 3) kamida k marta; 4) ko‘pi bilan k marta ro‘y berish ehtimoli quyidagi formulalar orqali hisoblash qulay.

$$a)P_n(0)+P_n(1)+....+P_n(k-1);$$
$$b)P_n(k+1)+P_n(k+1)+....+P_n(n);$$
$$v)P_n(k)+P_n(k+1)+....+P_n(n);$$
$$g)P_n(0)+P_n(1)+....+P_n(k).$$

3-masala. Ikki teng kuchli shaxmatchi shaxmat o‘ynashmoqda: Shaxmatchilarning to‘rt partiyadan ikkitasini yutish ehtimoli ko‘proqmi yoki olti partiyadan uchtasini yutish ehtimoli ko‘proqmi? (durrang natijalar hisobga olinmaydi)

Yechilishi. Teng kuchli shaxmatchilar o‘ynashmoqda: shu sababli partiyani yutish ehtimoli $p=\frac{1}{2}$, demak partiyani yutqazish ehtimoli q ham $\frac{1}{2}$ ga teng. Hamma partiyalarda yutish ehtimoli o‘zgarmas va partiyalarni qaysi tartibda yutishning farqi yo‘qligi sababli Bernulli formulasini qo‘llash mumkin. To‘rt partiyadan ikki partiyani yutish ehtimolini topamiz:

$$P_4(2)=C_4^2p^2q^2=\frac{4!}{2!\ 2!}\ \frac{1}{2}^{\ 2}\ \frac{1}{2}^{\ 2}=\frac{6}{16}.$$

Endi olti partiyadan uch partiyani yutish ehtimolini topamiz;

$$P_6(3)=C_6^3p^3q^3=\frac{6!}{3!\,3!}\;{\frac{1}{2}}^{3}\;{\frac{1}{2}}^{3}=\frac{5}{16}.$$

Yuqoridagi natijalarni taqqoslasak $P_4(2)>P_6(3)$ bo‘lgani uchun olti partiyadan uchtasini yutishdan ko‘ra to‘rt partiyadan ikkitasini yutish ehtimoli kattaroq.

4-masala. Tanga 5 marta tashlandi, gerbli tomon:

1) ikki martadan kam tushishi ehtimolini toping.

2) kamida ikki marta tushishi ehtimolini toping.

Yechish:

$$1)\; p=P_5(0)+P_5(1)=\frac{5!}{0!\,5!}\;{\frac{1}{2}}^{0}\;{\frac{1}{2}}^{5-0}+\frac{5!}{1!\,4!}\;{\frac{1}{2}}^{1}\;{\frac{1}{2}}^{5-1}=1\;1\;\frac{1}{32}+5\;\frac{1}{2}\;\frac{1}{16}=\frac{6}{32}=\frac{3}{16}$$

$$2)\; q=1-\;P_5(0)+P_5(1)\;=1-\;\frac{5!}{0!\,5!}\;{\frac{1}{2}}^{0}\;{\frac{1}{2}}^{5-0}+\frac{5!}{1!\,4!}\;{\frac{1}{2}}^{1}\;{\frac{1}{2}}^{5-1}\;=1-\frac{3}{16}=\frac{13}{16}$$

5-masala. Agar bitta sinovda A hodisaning ro‘y berish ehtimoli $0,4$ ga teng bo‘lsa, u holda to‘rtta erkli sinovda A hodisaning kamida uch martasida ro‘y berish ehtimolini toping.

Yechish: Masala shartiga ko‘ra $p=0,4$ va $q=1-p=1-0,4=0,6$ ga tengligini bilamiz.

Masala yechimini $P_n(k)=C_n^k\;p^k\;q^{n-k}$ Bernulli fo‘rmulasidan foydalanib hisoblaymiz.

$$P_4(3)+P_4(4)=\frac{4!}{3!\,1!}\;0,4^3\;\;0,6^{4-3}+\frac{4!}{4!\;0!}\;0,4^4\;\;0,6^{4-4}=0,1792.$$

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