Alimjonova Gulnozaxon Isroiljon qizi
Farg'ona politexnika instituti "Oliy matematika" kafedrasi katta o'qituvchisi
Telefon raqami: +998 997990207
Orcid: https://orcid.org/0000-0003-0280-6545
E-mail: alimjonovagulnozaxon@gmail.com

IKKINCHI TARTIBLI XUSUSIY HOSILALI GIPERBOLIK TIPDAGI TENGLAMA UCHUN RIMAN USULI

Annotatsiya: Psevdoparabolik tenglamalarning ko'p xususiyatlari giperbolik tipdagi tenglamalarga o'xshash bo'lganligi sababli ular uchun qo'yilgan masalalarga Riman usulini qo'llash mumkin. Shu sababli maqolada Riman funksiyasining tushunchalari haqida ma'lumot beramiz.

Kalit so'zlar: Riman funksiya, differensial tenglama, giperbolik tip, xususiy hosila, operator, integral.

МЕТОД РИМАНА ДЛЯ УРАВНЕНИЯ ГИПЕРБОЛИЧЕСКОГО ТИПА С ЧАСТНОЙ ПРОИЗВОДНОЙ ВТОРОГО ПОРЯДКА

Аннотация: Поскольку многие свойства псевдопараболических уравнений аналогичны уравнениям гиперболического типа, метод Римана можно применить к поставленным перед ними задачам. Поэтому в статье мы предоставим информацию о понятиях функции Римана.

Ключевые слова: Функция Римана, дифференциальное уравнение, гиперболического типа, специальная производная, оператор, интеграл.

RIEMANN METHOD FOR HYPERBOLIC EQUATION WITH SECOND ORDER PARTIAL DERIVATIVE

Annotation: Since many properties of pseudoparabolic equations are similar to equations of hyperbolic type, the Riemann method can be applied to the problems posed for them. Therefore, in the article, we will provide information about the concepts of the Riemann function.

Keywords: Riemannian function, differential equation, hyperbolic type, special derivative, operator, integral.

Ma'lumki, ikkinchi tartibli ikki o'zgaruvchili xususiy hosilali differensial tenglamaning koeffitsientlari yetarli umumiy shartlarni qanoatlantirganda, uni

$$Lu \equiv \frac{\partial^2 u}{\partial x \partial y} + a(x,y)\frac{\partial u}{\partial x} + b(x,y)\frac{\partial u}{\partial y} + c(x,y)u = f(x,y) \tag{1.1}$$

kanonik ko'rinishga keltirish mumkin. Agar (1.1) tenglamaning a va b koeffitsientlarini differensiallanuvchi deb hisoblasak, L operatorga qo'shma bo'lgan operator

$$M\upsilon \equiv \frac{\partial^2 \upsilon}{\partial x \partial y} - \frac{\partial}{\partial x}(a\upsilon) - \frac{\partial}{\partial y}(b\upsilon) + c\upsilon$$

ko‘rinishda yoziladi.

Riman usuli

Agar $a_x(x,y)$, $b_y(x,y)$ va $c(x,y)$ funksiyalar uzluksiz bo‘lsa,

$$Lu \equiv u_{xy} + au_x + bu_y + cu = f \qquad (1.1)$$

tenglama uchun $R(x_1, y_1; x, y)$ Riman funksiyasi mavjud va bu funksiya x_1, y_1 argumentlar bo‘yicha $Lu = 0$ tenglamani, x, y argumentlar buyicha qo‘shma $M\upsilon = 0$ tenglamani qanoatlantiradi.

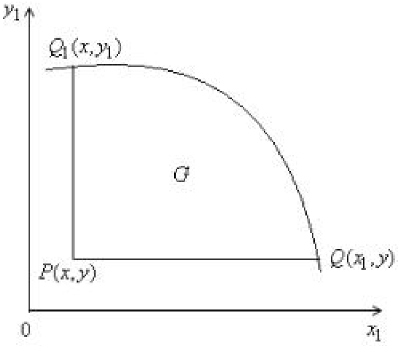
Riman funksiyasining bu va boshqa xossalaridan foydalanib, (1.1) tenglama uchun umumiy qo‘yilgan Koshi va Gursa masalalari yechimini topish mumkin. Bu usul Riman usuli deb ataladi.

Shunday qilib, (1.1) tenglamani qaraymiz. Uzluksiz egrilikka ega bo‘lgan ochiq Jordan chizig‘ini δ bilan belgilaymiz. Bu chiziq shunday xossaga ega bo‘lsinki, o‘zining hech bir nuqtasida (1.1) tenglamaning xarakteristikalari bilan urinishga ega bo‘lmasin. l esa δ da berilgan vektor bo‘lib, δ ning urinmasi bilan hech qanday nuqtada ustma-ust tushmasin.

1. Koshi masalasi. (1.1) tenglamaning

$$u\big|_{\delta} = \varphi, \quad \frac{\partial u}{\partial l}\bigg|_{\delta} = \psi \qquad (1.14)$$

boshlang‘ich shartlarni qanoatlantiruvchi regulyar yechimi topilsin, bu yerda φ va ψ mos ravishda ikki marta va bir marta uzluksiz differensiallanuvchi berilgan funksiyalardir.



Bu masala yechimini topish uchun δ chiziqda yotmaydigan ixtiyoriy $P(x,y)$ nuqta olaylik. $P(x,y)$ nuqtadan chiquvchi $x_1 = x$, $y_1 = y$ xarakteristikalar δ egri chiziq bilan Q_1 va Q nuqtalarda kesishadi deb faraz qilamiz.

RQ, RQ$_1$ to‘g‘ri chiziqlar va δ egri chiziqning *QQ*$_1$ qismi bilan chegaralangan sohani *G* orqali belgilab olamiz (1-chizma).

G sohada ixtiyoriy ikki marta uzluksiz differensiallanuvchi $u(x_1, y_1)$ va $\upsilon(x_1, y_1)$ funksiyalar uchun quyidagi ayniyat o‘rinli bo‘ladi:

$$2(\upsilon Lu - u M\upsilon) = \frac{\partial}{\partial y_1}\left(\frac{\partial u}{\partial x_1}\upsilon - \frac{\partial \upsilon}{\partial x_1}u + 2bu\upsilon\right) + \frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial y_1}\upsilon - \frac{\partial \upsilon}{\partial y_1}u + 2au\upsilon\right) \quad (1.15)$$

Bu ayniyatni *G* soha bo‘yicha integrallab, Gauss-Ostrogradskiy formulasini qo‘llash natijasida

$$2\int_G (\upsilon Lu - uM\upsilon)dx_1\, dy_1 = \int_S \left(\frac{\partial u}{\partial y_1}\upsilon - \frac{\partial \upsilon}{\partial y_1}u + 2au\upsilon\right)dy_1 - \left(\frac{\partial u}{\partial x_1}\upsilon - \frac{\partial \upsilon}{\partial x_1}u + 2bu\upsilon\right)dx_1$$

tenglikni hosil qilamiz, bunda *S*–*G* sohaning chegarasi, ya’ni $PQ + QQ_1 + Q_1P$.

PQ da $dy_1 = 0$, PQ_1 da $dx_1 = 0$ bo‘lgani uchun avvalgi tenglik quyidagi ko‘rinishda yoziladi:

$$2\int_G (\upsilon Lu - uM\upsilon)\, dx_1 dy_1 =$$

$$= \int_{QQ_1}\left(\frac{\partial u}{\partial y_1}\upsilon - \frac{\partial \upsilon}{\partial y_1}u + 2au\upsilon\right)dy_1 - \left(\frac{\partial u}{\partial x_1}\upsilon - \frac{\partial \upsilon}{\partial x_1}u + 2bu\upsilon\right)dx_1 +$$

$$+ \int_{Q_1}^{P}\left(\frac{\partial u}{\partial y_1}\upsilon - \frac{\partial \upsilon}{\partial y_1}u + 2au\upsilon\right)dy_1 - \int_{P}^{Q}\left(\frac{\partial u}{\partial x_1}\upsilon - \frac{\partial \upsilon}{\partial x_1}u + 2bu\upsilon\right)dx_1 . \quad (1.16)$$

Bu ifodaning o‘ng tomonidagi ikkinchi va uchinchi integrallarda $u(x_1, y_1)$ funksiyaning hosilalari qatnashgan hadlarni bo‘laklab integrallab, ushbu

$$\int_{Q_1}^{P}\left(\frac{\partial u}{\partial y_1}\upsilon - \frac{\partial \upsilon}{\partial y_1}u + 2au\upsilon\right)dy_1 = (u\upsilon)\Big|_{Q_1}^{P} - 2\int_{Q_1}^{P} u\left(\frac{\partial \upsilon}{\partial y_1} - a\upsilon\right)dy_1 ,$$

$$\int_{P}^{Q}\left(\frac{\partial u}{\partial x_1}\upsilon - \frac{\partial \upsilon}{\partial x_1}u + 2bu\upsilon\right)dx_1 = (u\upsilon)\Big|_{P}^{Q} - 2\int_{P}^{Q} u\left(\frac{\partial \upsilon}{\partial x_1} - b\upsilon\right)dx_1 \quad (1.17)$$

tengliklarga ega bo‘lamiz.

(1.16) formulada $u(x_1, y_1)$ funksiya (1.1), (1.14) Koshi masalasining yechimi, $\upsilon(x_1, y_1)$ esa Riman funksiyasi, ya’ni

$$\upsilon(x_1, y_1) = \upsilon(P_1) = R(x_1, y_1, x, y) = R(P_1, P)$$

bo‘lsin deb hisoblaymiz.

U holda, δ egri chiziqqa R_1 nuqtadan o‘tkazilgan normalni n orqali belgilab, $dy_1 = \frac{dx_1}{dn} ds,\ dx_1 = -\frac{dy_1}{dn} ds$ formulalarni e’tiborga olsak, (1.16) dan (1.17) ga va Riman funksiyasining (1.7) xossalariga asosan

$$\begin{aligned} u(P) &= \frac{1}{2}u(Q)R(Q,P) + \frac{1}{2}u(Q_1)R(Q_1,P) - \\ &- \frac{1}{2}\int_{QQ_1}\left[\frac{\partial u(P_1)}{\partial N}R(P_1,P) - u(P_1)\frac{\partial R(P_1,P)}{\partial N}\right]ds - \\ &- \frac{1}{2}\int_{QQ_1}\left[a(P_1)\frac{dx_1}{\partial n} + b(P_1)\frac{dy_1}{\partial n}\right]R(P_1,P)u(P_1)ds + \\ &+ \int_G f(P_1)R(P_1,P)dx_1\,dy_1 \end{aligned} \quad (1.18)$$

formulani hosil qilamiz, bunda $\frac{\partial}{\partial N} = \frac{dx_1}{dn}\frac{\partial}{\partial y_1} + \frac{dy_1}{dn}\frac{\partial}{\partial x_1}$.

(1.14) boshlang‘ich shartlarga asosan (1.18) formuladagi $\frac{\partial u}{\partial N}$ ni hamma vaqt bir qiymatli aniqlab olishimiz mumkin. (1.18) formula bilan aniqlangan $u(x,y)$ funksiyaning (1.1) tenglamani qanoatlantirishini tekshirib ko‘rish qiyin emas.

Shunday qilib, (1.18) formula (1.1), (1.14) Koshi masalasining yechimidan iboratdir. (1.18) formulani hosil qilish jarayonidan, bu masala yechimining yagonaligi va turg‘unligi ham kelib chiqadi.

2. Gursa masalasi. (1.1) tenglamaning $u(x, y_0) = \varphi(x),\ \ u(x_0, y) = \psi(y)$ shartlarni qanoatlantiruvchi regulyar yechimi topilsin, bu yerda $\varphi(x)$ va $\varphi(x)$ - uzluksiz differensiallanuvchi hamda $\varphi(x_0) = \psi(y_0)$ shartni qanoatlantiruvchi berilgan funksiyalar.

Bu masala yechimining ixtiyoriy $P(x, y)$ nuqtadagi qiymatini topaylik. Faraz qilaylik, $u(x,y) -$ Gursa masalasining yechimi, $R(x_1, y_1; x, y)$ esa (1.10) tenglamaning Riman funksiyasi bo‘lsin. U holda, ular uchun (1.10) tenglik o‘rinli. (1.10) da $u(x, y_0),\ u(x_0, y)$ va $Lu(x, y)$ ni mos ravishda $\varphi(x),\ \psi(y)$ va $f(x, y)$ ga almashtirsak, Gursa masalasi yechimini aniqlovchi

$$u(x,y)=R(x,y_0;x,y)\varphi(x)+R(x_0,y;x,y)\psi(y)-$$
$$-R(x_0,y_0;x,y)\varphi(x_0)+$$
$$+\int_{x_0}^{x}\left[b(x_1,y_0)R(x_1,y_0;x,y)-\frac{\partial}{\partial x_1}R(x_1,y_0;x,y)\right]\varphi(x_1)dx_1+$$
$$+\int_{y_0}^{y}\left[a(x_0,y_1)R(x_0,y_1;x,y)-\frac{\partial}{\partial y_1}R(x_0,y_1;x,y)\right]\psi(y_1)dy_1-$$
$$-\int_{x_0}^{x}dx_1\int_{y_0}^{y}R(x_1,y_1;x,y)f(x_1,y_1)dy_1$$

formula kelib chiqadi. Bu formulani hosil qilish jarayonidan yechimning yagonaligi ham kelib chiqadi. Masala yechimining turg'unligini ko'rsatish qiyinchilik tug'dirmaydi.

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