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Osiyo xalqaro universiteti, "Umumtexnik fanlar" kafedrasi o'qituvchisi

RIMAN-LIUVILL KASR TARTIBLI HOSILASINING ANIQLANISHI

Annotatsiya. Ushbu maqolada Riman-Liuvill kasr tartibli hosilasining aniqlanishi tadqiq etilgan, erishilgan natijalar keltirilgan.

Kalit so'zlar: Riman-Liuvil kasr hosila, kasr hosilaning mavjudligi, mavjudligning yetarli belgisi.

Yurtimiz Mustaqillikka erishganidan so'ng yoshlarimiz jahon arenalarida sport va fanning turli tarmoqlarida yuqori natijalarga erishib,yurtimiz sharafini butun dunyoga tanitib kelmoqda. Ularga jahon andozalari darajasida bilim berish,fan bilan ishlab chiqarish orasidagi munosabatni mustahkamlash, o'z sohasining yetuk mutaxasisi bo'lib yetishishi bugungi kunning dolzarb vazifalari hisoblanadi. Buning isbotini qabul qilinayotgan ta'limga oid qonun va qarorlarda ham yaqqol ko'rishimiz mumkin. Xususan matematika sohasida ko'plab sezilarli ishlar amalga oshirilmoqda. Bunga yaqqol misol sifatida O'zbekiston Respublikasi Prezidentining 07.05.2020 yildagi Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risidagi PQ-4708-sonli qarori hamda 08.10.2019 yildagi O'zbekiston Respublikasi oliy ta'lim tizimini 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to'g'risidagi O'zbekiston Respublikasi Prezidentining PF-5847-son farmonini keltirishimiz mumkin.

"Ta'lim to'g'risida"gi qonunda ta'lim tizimini yanada mukammallashtirish, sifatli ta'lim xizmatlari salohiyatini ko'tarish, mehnat bozorining zamonaviy ehtiyojlaridan kelib chiqqan holda yuqori malakali kadrlar tayyorlash siyosatini amalga oshirish; ta'lim muassasalarini bunyod etish va rekonstruksiya qilish va kapital ta'mirlash, zamonaviy o'quv-laboratoriya jihozlari, kompyuter texnikasi va o'quv uslubiy qo'llanmalar bilan jihozlash orqali ularning moddiy texnik bazasini mustahkamlash bo'yicha maqsadli chora tadbirlar amalga oshirish;

Ilmiy-tadqiqot va innovatsion faoliyatni rag'batlantirish, fan va innovatsion yutuqlarni amaliyotda joriy etishning samarali mexanizmlarini yaratish, oliy o'quv yurtlari va ilmiy-tadqiqot instutlari huzurida ixtisoslashtirilgan ilmiy-tadqiqot va tajriba labaratoriyalari, yuqori texnalogiyalari markazlari va texnoparklar tashkil etish to'g'risida muhim vazifalar belgilab berilgan.

Keyingi yillarda kasr hisobi tarmog'i matematika, fizika, gemistry, muhandislik va iqtisodyot va ijtimoiy fanlar kabi bir qancha sohalarda tadqiqotchilarning qiziqishini uyg'otdi. Bu sohadagi izlanishlar 19-asrda Liuvill, Rimann, Leybnits, Kaputo va boshqa olimlar tomonidan qilingan va o'rganilgan.

Tarif 1. $[a, b]$ kesmada berilgan $f(x)$ funksiya uchun quyidagi formulalar

$$(D_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_a^x \frac{f(t)dt}{(x-t)^{\alpha}} \qquad (1)$$

$$(D_{b-}^{\alpha} f)(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_x^b \frac{f(t)dt}{(t-x)^{\alpha}}, \quad 0< \alpha < 1 \qquad (2)$$

Riman-Liuvillning α tartibli kasr hosilasini ifodalaydi va bu hosilalar mos ravishda chap tomonli (1)ga va o'ng tomonli (2)ga deb aytiladi. Belgilab qo'yamiz kasr integrallar aniqlangan $\alpha > 0$ har qanday tartibi uchun,kasrli hosilalar esa hozircha faqat $0< \alpha < 1$ tartibi uchun $\alpha \geq 1$ tartibining kasrli hosilalarini aniqlashdan oldin, kasrli hosilalarning mavjudligining oddiy yetarli belgisini beramiz.

Lemma 1. Agar $f(x) \in AC([a, b])$ bo'lsa unda $f(x)$ funksiyani qariyb hamma yerda $(D_{a+}^{\alpha} f)(x)$ va $(D_{b-}^{\alpha} f)(x), 0 < \alpha < 1$, va shu bilan birga
$(D_{a+}^{\alpha} f)(x) \in L_1(a, b)$ va $(D_{b-}^{\alpha} f)(x) \in L_1(a, b)$ hosilalarga ega bo'ladi va ularni quyidagi ko'rinishda tasvirlash mumkin

$$(D_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(1-\alpha)} \left[\frac{f(a)}{(x-a)^{\alpha}} + \int_a^x \frac{f'(t)dt}{(x-t)^{\alpha}}\right] \qquad (3)$$

$$(D_{b-}^{\alpha} f)(x) = \frac{1}{\Gamma(1-\alpha)} \left[\frac{f(b)}{(b-x)^{\alpha}} + \int_x^b \frac{f'(t)dt}{(t-x)^{\alpha}}\right] \qquad (4)$$

Isbot. (3) va (4) formulalar lemma 1 ning natijasidan hosil bo‘ladi. $D^{\alpha}_{a+} f \in L_1(a,b)$ $\int_a^b |(D^{\alpha}_{a+} f)(x)|\, dx$ integralga aylanadi. Bizda bor

$\int_a^b |(D^{\alpha}_{a+} f)(x)|\, dx= \frac{1}{\Gamma(1-\alpha)} \int_a^b |\frac{f(a)}{(x-a)^{\alpha}} + \int_a^x \frac{f'(t)dt}{(x-t)^{\alpha}}|\, dx \le$

$\le \frac{1}{\Gamma(1-\alpha)} [\frac{|f(a)|(b-a)^{1-\alpha}}{1-\alpha} + \int_a^b dx \int_a^x \frac{|f'(t)|dt}{(x-t)^{\alpha}}]= \frac{1}{\Gamma(1-\alpha)} [\frac{|f(a)|(b-a)^{1-\alpha}}{1-\alpha} +$

$+\int_a^b |f'(t)|dt \int_t^b \frac{dx}{(x-t)^{\alpha}}]= \frac{1}{\Gamma(2-\alpha)}[|f(a)|(b-a)^{1-\alpha} + \int_a^b |f'(t)|\,(b-t)^{1-\alpha}dt]$

$f'(t)$ va $(b-t)^{1-\alpha}$ funksiyalar [a,b] segmentda ajralmas, $|f'(t)| \ge 0$, $(b-t)^{1-\alpha}$ uchun haqli ravishda $0 \le (b-t)^{1-\alpha} \le (b-a)^{1-\alpha}$, shuning uchuno‘rtacha qiymat teoremasi bo‘yicha bunday M son mavjud va

$0 \le M \le (b-a)^{1-\alpha}$, $\int_a^b |f'(t)|(b-t)^{1-\alpha}\, dt = M\int_a^b |f'(t)|dt$

munosabat o‘rinli. Quyidagi munosabatni tekshiraylik

$$\int_a^b |(D^{\alpha}_{a+} f)(x)|\, dx = \frac{1}{\Gamma(2-\alpha)}[|f(a)|(b-a)^{1-\alpha} + M\int_a^b |f'(t)|\, dt] < \infty,$$

bu esa $f(x) \in AC([a,b])$ anglatadi. Yuqoridagi munosabatlardan $f'(x) \in L_1(a,b)$ $\int_a^b |f'(t)|dt < \infty$ kelib chiqadi. Lemma isbotlandi.

Biz $\alpha \ge 1$ tartibning kasrli hosilalariga o‘tamiz. Quyidagilardan foydalanamiz:

$[\alpha] -$ α ning butun qismi;

$\{\alpha\} -$ α ning kasr qismi;

ta’rif bo‘yicha $0 \le \alpha < 1$ va $\alpha = [\alpha] + \{\alpha\}$.

Agar α –butun son bo‘lsa α ning kasr tartibli hosilasi oddiy hosilani anglatadi:

$D^{\alpha}_{a+} = \left(\frac{d}{dx}\right)^{\alpha}$, $D^{\alpha}_{b-} = \left(-\frac{d}{dx}\right)^{\alpha}$, $\alpha = 1,2,3\ldots.$

Agar α –butun son bo‘lmasa D^{α}_{a+} va D^{α}_{b-} larni quyidagicha aniqlaymiz

$(D^{\alpha}_{a+} f)(x)= \left(\frac{d}{dx}\right)^{[\alpha]} (D^{\{\alpha\}}_{a+} f)(x)= \left(\frac{d}{dx}\right)^{[\alpha]-1} (I^{1-\{\alpha\}}_{a+} f)(x),$

$(D^{\alpha}_{b-} f)(x)= \left(-\frac{d}{dx}\right)^{[\alpha]} (D^{\{\alpha\}}_{b-} f)(x) = \left(-\frac{d}{dx}\right)^{[\alpha]+1} (I^{1-\{\alpha\}}_{b-} f)(x)$

Ta’rif 2. $f(x),\ x \in [a,b]$ uchun

$$(D^{\alpha}_{a+} f)(x)= \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^{n} \int_a^x \frac{f(t)dt}{(x-t)^{\alpha-n+1}} \quad (5)$$

$$(D^{\alpha}_{b-} f)(x)= \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^{n} \int_x^b \frac{f(t)dt}{(t-x)^{\alpha-n+1}} \quad (6)$$

$n = [\alpha] + 1$, $\alpha > 0$, (5) va (6) formulalar mos ravishda. Riman-Liuvillning α kasr tartibli o‘ng va chap hosilasi deb nomlanadi. (5) va (6) hosilalarning mavjudligi uchun yetarli shart shundan iboratki $\int_a^x \frac{f(t)dt}{(x-t)^{\{\alpha\}}}$ integral $AC^{[\alpha]}([a,b])$ sinfga tegishli. Ushbu shartni qanoatlantirish uchun

$$f(x) \in AC^{[\alpha]}([a,b]) \quad (7)$$

Teorema 2. $\alpha \ge 0$ va $f(x) \in AC^{n}([a,b])$, $n = [\alpha] + 1$. U holda $D^{\alpha}_{a+} f$ deyarli hamma joyda mavjud bo‘lib, ularni quyidagicha ifodalash umkin

$$(D^{\alpha}_{a+} f)(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(1+k-\alpha)}(x-a)^{k-\alpha} + \frac{1}{\Gamma(n-\alpha)}\int_a^x \frac{f^{(n)}(t)dt}{(x-t)^{\alpha-n+1}} \quad (8)$$

Isbot. $f(x) \in AC^{n}$, keyin (8) quyidagicha isbotlaymiz.

$(D^{\alpha}_{a+} f)(x)= \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^{n} \int_a^x \frac{f(t)dt}{(x-t)^{\alpha-n+1}} =$

$= \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n \int_a^x \frac{dt}{(x-t)^{\alpha-n+1}}[\sum_{k=0}^{n-1} c_k (t-a)^k + \frac{1}{(n-1)!}(t-y)^{n-1}$ (y) dy]=

$= \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n [\sum_{k=0}^{n-1} c_k (t-a)^k (x-t)^{n-\alpha-1}dt +$

$+\frac{1}{(n-1)!}\int_a^x \frac{dt}{(x-t)^{\alpha-n+1}} \int_a^t (t-y)^{n-1}\varphi(y)dy].$

Biz $\int_a^x (t-a)^k (x-t)^{n-\alpha-1}dt$ integralni hisoblaymiz, $t = a + \tau(x-a)$ almashtirish bajaramiz:

$\int_a^x (t-a)^k (x-t)^{n-\alpha-1}dt = (x-\alpha)^{n+k-\alpha} \quad \int_0^1 \tau^k (1+\tau)^{n-\alpha-1}d\tau=$

$$= (x-\alpha)^{n+k-\alpha}B(k+1, n-\alpha) = (x-\alpha)^{n+k-\alpha}\frac{\Gamma(k+1)\Gamma(n-\alpha)}{\Gamma(n-\alpha+k+1)}$$

$\int_a^x (x-t)^{n-\alpha-1} dt \int_a^t (t-y)^{n-1}\varphi(y)dy$ integralga avval biz Dirixle formulasini qo‘llaymiz va keyin $t = y + \tau(x-y)$ almashtirish yordamida ichki integralni hisoblaymiz.

$$\int_a^x (x-t)^{n-\alpha-1} dt \int_a^t (t-y)^{n-1}\varphi(y)dy =$$

$= \int_a^x \varphi(y)dy \int_y^x (x-t)^{n-\alpha-1}(t-y)^{n-1} dt = \frac{\Gamma(n)\Gamma(n-\alpha)}{\Gamma(2n-\alpha)}\int_a^x \varphi(y)(x-y)^{2n-\alpha-1}dy$

Shuning uchun $(D_{a+}^{\alpha} f)(x)$ uchun bizda bo‘ladi

$(D_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n [\sum_{k=0}^{n-1} c_k (x-\alpha)^{n+k-\alpha}\frac{\Gamma(k+1)\Gamma(n-\alpha)}{\Gamma(n-\alpha+k+1)} +$

$+\frac{1}{(n-1)!}\frac{\Gamma(n)\Gamma(n-\alpha)}{\Gamma(2n-\alpha)}\int_a^x \varphi(y)(x-y)^{2n-\alpha-1}dy] =$

$= \frac{1}{\Gamma(n-\alpha)}[\sum_{k=0}^{n-1} c_k \left(\frac{d}{dx}\right)^n (x-\alpha)^{n+k-\alpha}\frac{\Gamma(k+1)\Gamma(n-\alpha)}{\Gamma(n-\alpha+k+1)} +$

$+ \frac{1}{(n-1)!}\frac{\Gamma(n)\Gamma(n-\alpha)}{\Gamma(2n-\alpha)}\int_a^x \varphi(y)\left(\frac{d}{dx}\right)^n (x-y)^{2n-\alpha-1}dy]$

………..

$\left(\frac{d}{dx}\right)^n (x-\alpha)^{n+k-\alpha} = (n+k-\alpha)(n+k-\alpha-1)\dots(k-\alpha+1)(x-a)^{k-\alpha}=$

$=(k-\alpha+1)_n (x-a)^{k-\alpha} = \frac{\Gamma(k-\alpha+n+1)}{\Gamma(k-\alpha+1)}(x-\alpha)^{k-\alpha},$

$\left(\frac{d}{dx}\right)^n (x-y)^{2n-\alpha-1} = \frac{\Gamma(2n-\alpha)}{\Gamma(n-\alpha)}(x-y)^{n-\alpha-1},$

…………………………

$(D_{a+}^{\alpha} f)(x) = \sum_{k=0}^{n-1} c_k (x-\alpha)^{k-\alpha}\frac{\Gamma(k-1)}{\Gamma(k-\alpha+1)} + \frac{1}{\Gamma(n-\alpha)}\int_a^x \varphi(y)(x-y)^{n-\alpha-1}dy.$

……..$c_k = \frac{f^{(k)}(a)}{k!}$ va $\varphi(y) = f^{(n)}(y)$

$(D_{a+}^{\alpha} f)(x) = \sum_{k=0}^{n-1}\frac{f^{(k)}(a)}{k!}\frac{\Gamma(k+1)(x-\alpha)^{k-\alpha}}{\Gamma(k-\alpha+1)} + \frac{1}{\Gamma(n-\alpha)}\int_a^x \frac{f^{(n)}(y)}{(x-y)^{\alpha-n+11}}dy=$

$=\sum_{k=0}^{n-1}\frac{f^{(k)}(a)}{\Gamma(k-\alpha+1)}(x-\alpha)^{k-\alpha} + \frac{1}{\Gamma(n-\alpha)}\int_a^x \frac{f^{(n)}(y)}{(x-y)^{\alpha-n+11}}dy.$

Teorema isbotlandi.

Xulosa

Shunday qilib, ushbu maqolada Riman-Liuvill kasr tartibli hosilasining aniqlanishi tadqiq qilindi.

Kasr tartibli differensial tenglamalar sistemasini o’rganish quyidagi imkoniyatlarni taqdim etadi:

1) Termodinamika masalalarini yechish;

2) Geologiyada yer osti qatlamlaridagi tektonik harakatlarni tadqiq qilish;

3) Termoelastiklik masalalarini modellarini qurish.

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